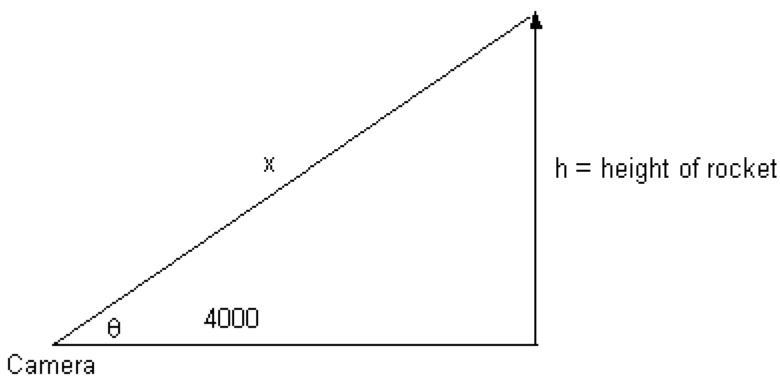
Math 131, Fall 2004
Discussion Section 9

1. A t.v. camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Assume the rocket rises vertically and that its speed is 600 ft/s when it has risen 3000 ft.



a) How fast is the distance from the t.v. camera to the rocket increasing at that moment?

Let x be the distance from the camera to the rocket, and h the height of the rocket. We have $x^2 = h^2 + 4000^2$, so $2x\frac{dx}{dt} = 2h\frac{dh}{dt}$, so $x\frac{dx}{dt} = h\frac{dh}{dt}$.

When $h = 3000$, $x^2 = 3000^2 + 4000^2$, which gives $x = 5000$. We are told that at that instant, $\frac{dh}{dt} = 600$ ft/s. So, at that instant $5000\frac{dx}{dt} = 3000(600)$ and $\frac{dx}{dt} = \frac{3000(600)}{5000} = \frac{1800}{5} = 360$ ft/sec

b) If the t.v. camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that moment?

$\tan\theta = \frac{h}{4000}$, so $h = 4000\tan\theta$. Then $\frac{dh}{dt} = 4000\sec^2\theta \cdot \frac{d\theta}{dt}$. When $h = 3000$, $\tan\theta = \frac{3}{4}$. Since $1 + \tan^2\theta = \sec^2\theta$, we get $\sec^2\theta = 1 + (\frac{3}{4})^2 = 1 + \frac{9}{16} = \frac{25}{16}$. Then $600 = 4000 \cdot \frac{25}{16} \cdot \frac{d\theta}{dt}$, so $\frac{d\theta}{dt} = \frac{600}{4000} \cdot \frac{16}{25} = \frac{3}{20}\frac{16}{25} = \frac{12}{125} = 0.096$ radians/s.

2. Water is contained in a cylindrical tank. How fast does the water level drop when the tank is drained at a rate of 3 liters/sec ($= 3000$ cm^3/sec) ? (*The answer depends on the radius of the tank.*)

The water in the tank forms a cylinder with some unknown radius r and height h ($=$ the depth of the water.) Let's measure r and h in cm (to make the connection with liters easy). The volume of water at time t is $V = \pi r^2 h$ (cm^3). As the water drains, r remains constant and V changes at a rate $\frac{dV}{dt} = \; = -3\,L/\text{sec} = -3000$ cm^3/sec. Since $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$, we get that $\frac{dh}{dt} = \frac{dV/dt}{\pi r^2} = \frac{-3000}{\pi r^2}$ cm/sec

3. Consider the function $f(x) = e^x(x-2)$ defined on the interval $[-1, 3]$.

a) What are the (absolute) maximum and minimum values of f and where do they occur? Are there any local maxima or minima ?

$f'(x) = e^x + e^x(x-2) = e^x + xe^x - 2e^x = xe^x - e^x = e^x(x-1)$
The only critical number is where $f'(x) = 0$, that is, where $x = 1$.

Since f is continuous on the closed interval $[-1, 3]$, f has absolute max and min values, and these occur at the endpoints $-1, 3$ or at a critical point in $(-1, 3)$. Soe we need to test the values of f at $-1, 1, 3$. We get $f(-1) = -3e^{-1} \approx -1.1$, $f(1) = -e \approx -2.7183$, and $f(3) = e^3 \approx 20.0855$. Therefore

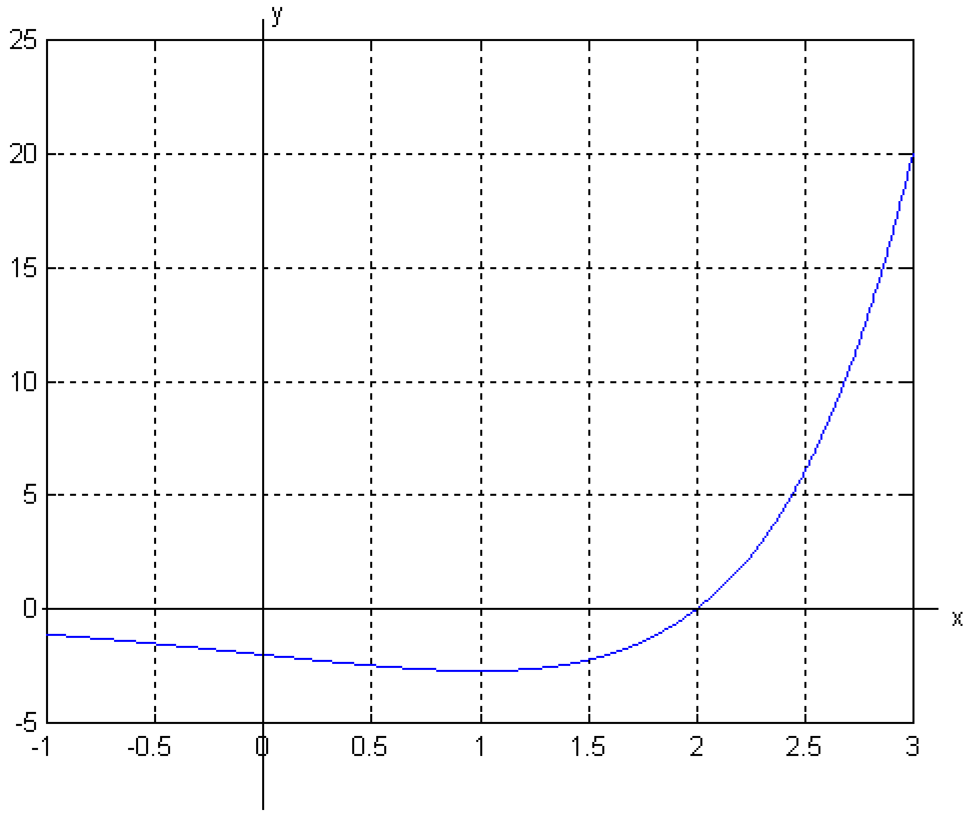
Absolute max value $= e^3 \approx 20.0855$, occuring at the right endpoint $x = 3$
Absolute min value $= -e \approx -2.7183$, occuring at $x = 1$

Since local maxima/minima do not occur at endpoints, the only possible candidate for local maxima/minima is at $x = 1$. Since there is an absolute min there (and it's not an endpoint), there's also a local min there.

b) Does $f(x)$ have any inflection points?

$f''(x) = e^x + e^x(x-1) = e^x + xe^x - e^x = xe^x$. $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$, so f switches concavity at x: there is an inflection point at $x = 0$.

c) Draw a reasonable sketch of the graph.



y
x
25
20
15
10
5
0
-5
-1
-0.5
0
0.5
1
1.5
2
2.5
3