Math 131 **Exam 4 (Final Exam) Solutions, Fall 2004**

The exam consists of 18 multiple choice questions (5 points each) and 10 true/false questions (1 point each), for a total of 100 points.

1. We begin with 20g of a certain radioactive isotope. As it decays, the amount remaining at time t (hrs) is $A = 20e^{-0.08t}$. What is its rate of decay at time $t = 8$? (*Round your answer to 2 decimal places.*)

A) -0.21 g/hr	B) -0.33 g/hr	C) -0.38 g/hr
D) -0.48 g/hr	E) -0.76 g/hr	**F) -0.84 g/hr**
G) -0.89 g/hr	H) -0.94 g/hr	I) -0.98 g/hr
J) -1.03 g/hr		

The rate of change of A is given by $\frac{dA}{dt} = 20e^{-0.08t}(-0.08)$
$= -\frac{8}{5}e^{0.08t}$. When $t = 8$, $\frac{dA}{dt} = -\frac{8}{5}e^{-0.64}$ g/hr ≈ -0.84 g/hr.

2. If $\int_0^7 g(x)\,dx = 20$, $\int_4^7 g(x)\,dx = 4$, and $\int_0^1 g(x)\,dx = 7$, then what is $\int_1^4 g(x)\,dx$?

A) 10	**B) 9**	C) 11	D) 7	E) 6
F) 16	G) 23	H) 13	I) 8	J) 27

$\int_0^7 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^4 g(x)\,dx + \int_4^7 g(x)\,dx$, so
$20 = 7 + \int_1^4 g(x)\,dx + 4$, and $\int_1^4 g(x)\,dx = 20 - 11 = 9$.

3. If $y = \sin^2(2x) + (x - \frac{\pi}{2})\tan(2x)$, what is the slope of the tangent line to the graph at $(0,0)$?

A) $-\frac{\pi}{2}$	B) $\frac{1}{2}$	C) $\frac{\pi}{4}$	D) $2 + \frac{\pi}{2}$	**E) $-\pi$**
F) 2π	G) 0	H) -1	I) 2	J) -2

At any point on the graph, the slope is given by
$\frac{dy}{dx} = 2\sin(2x)\cos(2x)\cdot 2 + (x - \frac{\pi}{2})\sec^2(2x)\cdot 2 + \tan(2x)$
$= 4\sin(2x)\cos(2x) + 2(x - \frac{\pi}{2})\sec^2(2x) + \tan(2x)$.
When $x = 0$, $\frac{dy}{dx} = 0 + 2(-\frac{\pi}{2})\sec^2(0) + 0 = -\pi$.

4. For a certain gas held inside a container, the pressure and volume are related by the equation $PV = 1$. Use a differential to estimate the change in pressure when the volume decreases from 10 cm^3 to 9.5 cm^3. (*Round your answer to 4 decimal places.*)

A) -0.0047 B) -0.0042 C) 0.0142 D) 0.0148
E) $\mathbf{0.0050}$ F) 0.0053 G) -0.1673 H) -0.1610
I) 0.5712 J) 5.1314

Since $P = \frac{1}{V}$, we have $\frac{dP}{dV} = -\frac{1}{V^2}$, so $dP = -\frac{1}{V^2}dV$.
If V changes from 10 to 9.5, then $dV = -0.5$, so
$dP = -\frac{1}{10^2}(-0.5) = \frac{1}{200} = 0.0050$
Observe that the relationship between P and V implies that P increases when V decreases, so you should have expected a positive value for dP. The actual change in pressure (rounded to 4 decimal places) is $\Delta P =$ new pressure $-$ old pressure
$= \frac{1}{9.5} - \frac{1}{10} \approx 0.0053$)

5. At time t, the velocity of a point moving along a line is $t^2(t^3+1)^7$ m/sec.
At time $t = -1$, the point is at position 0. What is its position when $t = 0$?

A) $\frac{1}{50}$ m B) 1 m C) $\frac{1}{2}$ m **D) $\mathbf{\frac{1}{24}}$ m** E) 2 m
F) $-\frac{1}{2}$m G) -1 m H) -2 m I) $\frac{5}{2}$ m J) $\frac{1}{8}$ m

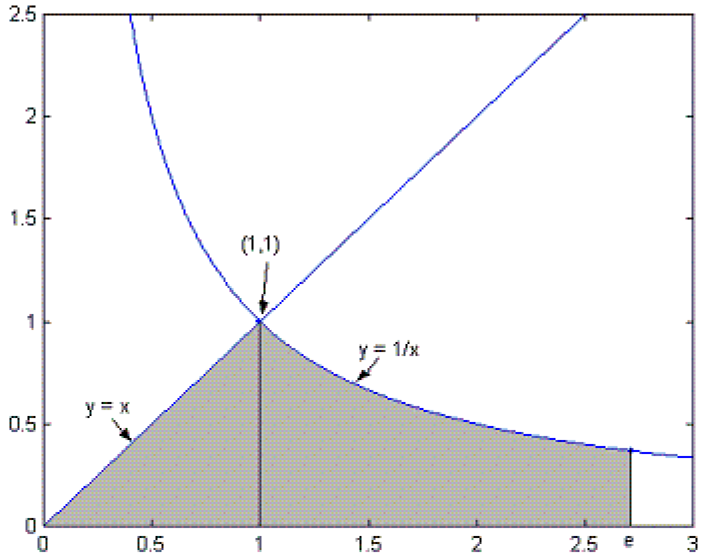
The position s is an antiderivative for the velocity: $s = \int t^2(t^3+1)^7\,dt$.
You might be able to get a formula for s by trial and error. However, the substitution $u = t^3+1,\ du = 3t^2\,dt$ gives $s = \int t^2(t^3+1)^7\,dt = \int \frac{1}{3}u^7\,du = \frac{u^8}{24} + C$
$= \frac{(t^3+1)^8}{24} + C$. When $t = -1$, we have $0 = s = 0 + C$, so $C = 0$. Therefore
$s = \frac{(t^3+1)^8}{24}$. When $t = 0$, $s = \frac{1}{24}$.

6. If $f(x) = \ln\left(\frac{(x^2+1)}{\sqrt[5]{\cos x}}\right)$, what is $f'(1)$?

A) 0 B) $\ln(2)$ C) $\tan(1)$ D) 0
E) $\mathbf{1 + \frac{1}{5}\tan(1)}$ F) $1 + \frac{1}{5}\sin(1)$ G) $\frac{1}{5}\sin(1)$
H) $\frac{2}{\sqrt[5]{\cos(1)}}$ I) $-\frac{1}{5}\ln\left(\frac{2}{\sin(1)}\right)$ J) $\ln(\tan(1))$

First we simplify: $f(x) = \ln\left(\frac{(x^2+1)}{\sqrt[5]{\cos x}}\right) = \ln(x^2+1) - \frac{1}{5}\ln(\cos x)$. Then
$f'(x) = \frac{2x}{x^2+1} - \frac{1}{5}\frac{(-\sin x)}{\cos x} = \frac{2x}{x^2+1} + \frac{1}{5}\tan x$. So $f'(1) = 1 + \frac{1}{5}\tan(1)$.

7. Find the shaded area:



A) $\mathbf{\frac{3}{2}}$ B) $\frac{2}{3}$ C) $\frac{5}{3}$ D) $\frac{7}{4}$ E) 2
F) $\ln 2$ G) $\ln(\frac{2}{3})$ H) 2 I) $\frac{5}{2}$ J) $\ln(3)$

The shaded area is (area of the triangle under $y = x$ above $[0,1]$) + (area under the graph of $y = \frac{1}{x}$ above $[1, e]$). The area of the triangle can be done by simple geometry, but here is the whole computation with integrals:
Area $= \int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx = \frac{x^2}{2}\big|_0^1 + \ln|x|\,\big|_1^e = (\frac{1}{2} - 0) + (\ln e - \ln 1)$
$= \frac{1}{2} + \ln e = \frac{1}{2} + 1 = \frac{3}{2}.$

8. $\lim\limits_{h \to 0} \frac{1}{h}\left(\frac{2}{3h+4} - \frac{2}{4}\right)$ is which of the following ?

A) $f'(\frac{2}{3})$ where $f(x) = \ln(\frac{x}{4})$
B) $f'(0)$ where $f(x) = \frac{3x+4}{2x}$
C) $f'(2)$ where $f(x) = \frac{2}{3x+4}$
D) $f'(4)$ where $f(x) = \frac{2}{3x+4}$
E) $f'(0)$ where $f(x) = \frac{2}{x+4} + x$
F) $f'(2)$ where $f(x) = 2\ln(3x+4)$
G) $f'(3)$ where $f(x) = \frac{2}{x+4} + x$
H) $\boldsymbol{f'(0)}$**, where** $\boldsymbol{f(x) = \frac{2}{3x+4}}$
I) $f'(\frac{2}{3})$ where $f(x) = \frac{1}{x+4}$
J) $f'(3)$ where $f(x) = \frac{2}{3x+4}$

If $f(x) = \frac{2}{3x+4}$, then $f'(0) = \lim\limits_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim\limits_{h \to 0} \frac{f(h) - f(0)}{h}$
$= \lim\limits_{h \to 0} \frac{1}{h}\left(\frac{2}{3h+4} - \frac{2}{4}\right).$

9. What is the slope of the tangent line to $\cos y + \sqrt{3+x^2} = 2$ at the point $(1, \frac{\pi}{2})$?

A) 0	B) 1	**C) $\frac{1}{2}$**	D) $\frac{3}{4}$	E) 2
F) -1	G) $-\frac{2}{3}$	H) $-\frac{1}{4}$	I) -2	J) tangent is vertical

To find the slope at a point, we need to $\frac{dy}{dx}$. Implicit differentiation gives $(-\sin y)\frac{dy}{dx} + \frac{1}{2\sqrt{3+x^2}}(2x) = (-\sin y)\frac{dy}{dx} + \frac{x}{\sqrt{3+x^2}} = 0$. When $x = 1$ and $y = \frac{\pi}{2}$, we get $(-\sin\frac{\pi}{2})\frac{dy}{dx} + \frac{1}{\sqrt{3+1}} = 0$, so $\frac{dy}{dx} = \frac{1}{2}$.

10. What is the maximum value of $g(\theta) = \theta - \sin(2\theta)$ on the interval $[0, \pi]$?

A) $\frac{1}{2}$	B) $\frac{\sqrt{3}}{2}$	C) $\frac{\pi}{4} + \frac{\sqrt{3}}{2}$	D) π	**E) $\frac{5\pi}{6} + \frac{\sqrt{3}}{2}$**
F) $\frac{5\pi}{6} - \frac{1}{2}$	G) $\frac{\pi}{2} + 1$	H) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$	I) $\frac{5\pi}{6} + \frac{\sqrt{2}}{2}$	J) $\frac{3\pi}{4} + \frac{1}{2}$

g is a continuous function on a closed interval, so the Extreme Value Theorem guarantees that there is an (absolute) maximum value, and it must occur at either one of the endpoints $0, \pi$, or at a critical number for g between 0 and π. We find these critical numbers:

$g'(\theta) = 1 - 2\cos(2\theta) = 0$ gives $\cos(2\theta) = \frac{1}{2}$. The positive solutions for this equation are $2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \ldots$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \ldots$ Of these solutions, only $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are between 0 and π.

Then we test the value of g at each "candidate":

$$g(0) = 0$$
$$g(\tfrac{\pi}{6}) = \tfrac{\pi}{6} - \sin(\tfrac{\pi}{3}) = \tfrac{\pi}{6} - \tfrac{\sqrt{3}}{2} \approx -0.34$$
$$g(\tfrac{5\pi}{6}) = \tfrac{5\pi}{6} - \sin(\tfrac{5\pi}{3}) = \tfrac{5\pi}{6} + \tfrac{\sqrt{3}}{2} \approx 3.48$$
$$g(\pi) = \pi - \sin(2\pi) = \pi \approx 3.14$$

The maximum value of g is $\frac{5\pi}{6} + \frac{\sqrt{3}}{2}$.

11. Find $\int_0^{\pi/4} \sec^2 x + \cos x \, dx$.

A) 0	B) $\frac{\sqrt{3}}{2}$	C) $\frac{\sqrt{3}}{3}$	D) $\frac{\sqrt{2}}{2}$	E) $\frac{1+\sqrt{3}}{2}$
F) $\frac{2+\sqrt{2}}{2}$	G) $\frac{2+\sqrt{3}}{3}$	H) $3+\sqrt{2}$	I) 1	J) $\frac{1+\sqrt{3}}{3}$

$\int_0^{\pi/4} \sec^2 x + \cos x \, dx = (\tan x + \sin x)\big|_0^{\pi/4} = (1+\frac{\sqrt{2}}{2}) - (0+0) = \frac{2+\sqrt{2}}{2}$

12. The curve $\begin{cases} x = \sin t \\ y = \sin(t+\sin(t)) + t \end{cases}$ passes through $(0,0)$.

What is the slope of the tangent line at $(0,0)$?

A) 4	**B) 3**	C) 2	D) 1	E) 0
F) -1	G) -2	H) -3	I) -4	J) -5

The slope of the tangent at a point is given by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(\cos(t+\sin t))(1+\cos t)+1}{\cos t}$
At $(0,0)$ (corresponding to $t=0$), $\frac{dy}{dx} = \frac{(\cos(0+0))(1+\cos 0)+1}{\cos 0} = 3.$

13. The function $f(x) = ax^5 + 160x^2 + c$ has an inflection point at $(2, 580)$.
What are a and c?

A) $a=1,\ c=-92$	B) $a=2,\ c=-124$	C) $a=3,\ c=-156$
D) $a=1,\ c=-10$	E) $a=0,\ c=-60$	F) $a=-1,\ c=-28$
G) $a=-2,\ c=4$	H) $a=-3,\ c=36$	I) $a=-4,\ c=21$
J) $a=-2,\ c=1$		

$f'(x) = 5ax^4 + 320x$ and $f''(x) = 20ax^3 + 320$. Since there is an inflection point where $x=2$, we must have $f''(2) = 20a(2^3) + 320 = 0$, so $a = -2$. Therefore $f(x) = -2x^5 + 160x^2 + c$.
Since $f(2) = -2\cdot 2^5 + 160\cdot 2^2 + c = -64 + 640 + c = 576 + c = 580$, we get $c = 4$.

14. Since $\int_1^4 \frac{1}{x}dx = \ln(4)$, we can compute an approximate value for $\ln(4)$ by approximating the integral $\int_1^4 \frac{1}{x}dx$. A very rough approximation would be the midpoint approximation M_3. What is M_3 ? (*Round your answer to four decimal places.*)

A) 1.3325 B) 1.3422 C) 1.3476 D) 1.3522 **E) 1.3524**
F) 1.3601 G) 1.3678 H) 1.3742 I) 1.3863 J) 1.3977

$\Delta x = \frac{4-1}{3} = 1$ and the 3 subintervals for computing M_3 are $[1,2]$, $[2,3]$, and $[3,4]$.
The midpoints of the subintervals are $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ so
$M_3 = \frac{1}{3/2}\Delta x + \frac{1}{5/2}\Delta x + \frac{1}{7/2}\Delta x = \frac{2}{3} + \frac{2}{5} + \frac{2}{7} = \frac{142}{105} \approx 1.3524.$
(*Rounded to 4 decimal places, a calculator gives the value ln* $4 \approx 1.3863$.)

15. Suppose the interval $[1,3]$ is divided into n equal subintervals and the right endpoint x_i is chosen from each subinterval. What is the value of $\lim_{n\to\infty} \left(\sum_{i=1}^{n}(x_i^2 + \frac{1}{x_i^2}) \cdot \frac{2}{n}\right)$?

A) $\frac{1}{2}$ B) $\frac{17}{3}$ C) $\frac{35}{9}$ **D) $\frac{28}{3}$** E) $\frac{19}{4}$
F) $\frac{37}{2}$ G) $\frac{43}{9}$ H) $\frac{11}{4}$ I) $\frac{43}{3}$ J) $\frac{57}{4}$

If $f(x) = x^2 + \frac{1}{x^2}$, then $\sum_{i=1}^{n}(x_i^2 + \frac{1}{x_i^2}) \cdot \frac{2}{n}$ is just the right endpoint Riemann sum R_n.
Therefore $\lim_{n\to\infty} \left(\sum_{i=1}^{n}(x_i^2 + \frac{1}{x_i^2}) \cdot \frac{2}{n}\right) = \lim_{n\to\infty} R_n = \int_1^3 x^2 + \frac{1}{x^2}\, dx = (\frac{x^3}{3} - \frac{1}{x})\Big|_1^3$
$= (\frac{27}{3} - \frac{1}{3}) - (\frac{1}{3} - 1) = 10 - \frac{2}{3} = \frac{28}{3}.$

16. An investment is growing at a rate of $100e^{0.01t}$ (\$/yr). How much does its value increase between the second and third years? (*Round your answer to the nearest cent.*)

A) \$96.03 B) \$97.11 C) \$97.78 D) \$98.13 E) \$99.21
F) \$100.45 G) \$100.97 H) \$101.63 I) \$102.11 **J) \$102.53**

The total change in the value of the investment between $t = 2$ and $t = 3$ is
$\int_2^3$(rate of change of the value) $dt = \int_2^3 100e^{0.01t}\, dt = 10000e^{0.01t}\Big|_2^3$
$= 10000(e^{0.03} - e^{0.02}) \approx \$102.53.$

17. If $F(x) = (\sin x) \cdot \int_1^{3x} \frac{\sin t}{t}\, dt,$ what is $F'(\frac{\pi}{2})$?

A) 0	**B)** $-\frac{2}{\pi}$	C) π	D) $-\pi$	E) $\frac{\pi}{3}$
F) -1	G) $\frac{3\pi}{2}$	H) $-\frac{2}{3\pi}$	I) $\frac{3}{2}$	J) -2π

We need the Product Rule, the Fundamental Theorem of Calculus (Part I), and the Chain Rule:

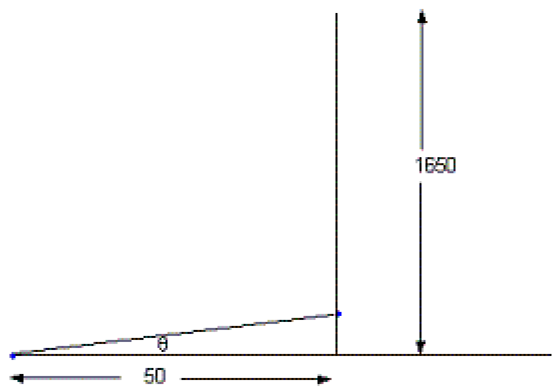
$F'(x) = (\sin x) \cdot \frac{d}{dx}(\int_1^{3x} \frac{\sin t}{t}\, dt) + (\int_1^{3x} \frac{\sin t}{t}\, dt) \cdot \frac{d}{dx} \sin x$

$= (\sin x)\frac{\sin 3x}{3x}(3) + (\cos x)\int_1^{3x} \frac{\sin t}{t}\, dt.$

So $F'(\frac{\pi}{2}) = (\sin\frac{\pi}{2})\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}(3) + (\cos\frac{\pi}{2})\int_1^{\frac{3\pi}{2}} \frac{\sin t}{t}\, dt$

$= (1)\frac{-1}{3\pi/2}(3) + (0)\int_1^{\frac{3\pi}{2}} \frac{\sin t}{t}\, dt = -\frac{2}{\pi}.$

18. A rock is dropped from the top of a cliff 1650 ft. high. A camera on the ground, 50 ft. away from the base of the cliff, stays focussed on the rock as it falls. How fast is the angle of elevation of the camera θ changing at the moment when the rock is 50 feet from the ground?



A) -1.2 rad/sec B) -0.34 rad/sec C) -8.2 rad/sec D) -2.5 rad/sec
E) -1.3 rad/sec F) -2.1 rad/sec **G) -3.2 rad/sec** H) -6.5 rad/sec
I) -4 rad/sec J) -5 rad/sec

Let $s =$ the height of the rock above the ground at time t. We want to find $\frac{d\theta}{dt}$ when $s = 50$.

Since $\tan\theta = \frac{s}{50}$, we get $\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{50}\frac{ds}{dt}$, so $\frac{d\theta}{dt} = \frac{ds/dt}{50\sec^2\theta}$.
When $s = 50$, $\theta = \frac{\pi}{4}$ and $\sec^2\frac{\pi}{4} = 2$, so at that moment $\frac{d\theta}{dt} = \frac{ds/dt}{100}$.

So we need to know $\frac{ds}{dt}$ when $s = 50$. For the falling rock, the acceleration $a(t) = -32$ ft/sec^2, so the velocity $v(t) = -32t + C$.
Since $v(0) = 0$, $v(t) = -32t$ ft/sec and that gives $s(t) = -16t^2 + D$ ft. Since $s(0) = 1650$, we have $s(t) = -16t^2 + 1650$.

Then $s = 50 = -16t^2 + 1650$ gives $16t^2 = 1600$, so $t = 10$ (sec). At that time $\frac{ds}{dt} = v = -32(10) = -320$ ft/sec.

Therefore at that time $\frac{d\theta}{dt} = \frac{ds/dt}{100} = -\frac{320}{100} = -3.2$ (rad/sec).

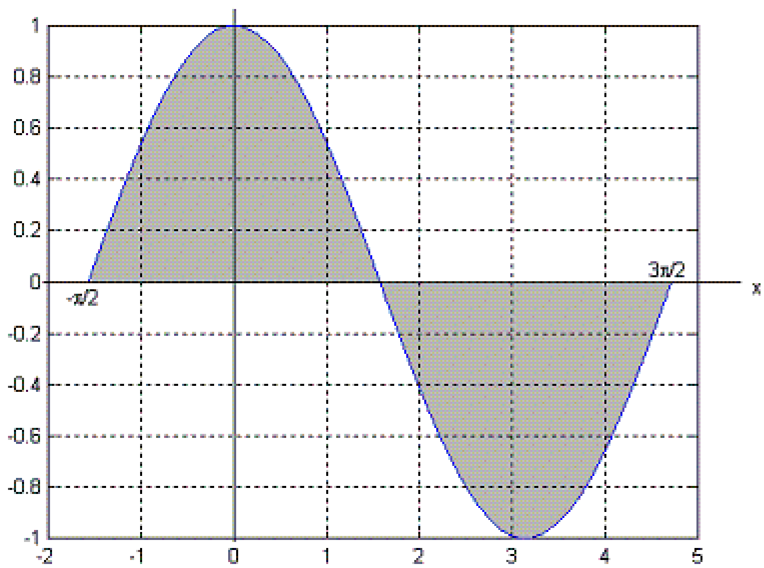
The following 10 questions are all true/false questions (1 point each)

19. The following integrals are correctly listed in order of increasing size:

$\int_{-\frac{\pi}{2}}^{0} \cos x \, dx < \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx < \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx$

A) True **B) False**

The first two integrals are positive and correctly ordered, but the last integral $= 0$.



20. Suppose a point is moving along a straight line with velocity $v(t)$. Its position at time $t = 0$ is s_0. Then at time t, its position is $s_0 + \int_0^t v(u) \, du$.

A) True B) False

$\int_0^t v(u) \, du = s(u)|_0^t = s(t) - s(0) = s(t) - s_0$, so
$s(t) = \int_0^t v(u) du + s_0$.

21. If $b \neq 0$, $\lim\limits_{x \to \infty} \frac{ax^3 + e^{-x}}{bx^3 + 1} = \frac{a}{b}$.

A) True B) False

$\lim\limits_{x \to \infty} \frac{ax^3 + e^{-x}}{bx^3+1} = \lim\limits_{x \to \infty} \frac{a + (e^{-x})/x^3}{b + 1/x^3}$. Since $\lim\limits_{x \to \infty} \frac{e^{-x}}{x^3} = \lim\limits_{x \to \infty} \frac{1}{x^3 e^x} = 0$,
$\lim\limits_{x \to \infty} \frac{a + (e^{-x})/x^3}{b + 1/x^3} = \frac{a + 0}{b + 0} = \frac{a}{b}$.

22. Suppose $y = f(x)$ is differentiable and has only one critical number, $x = 1$. If $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$, then $f(x)$ has an absolute minimum at $x = 1$.

A) True B) False

The derivative tells us that f is decreasing to the left of 1 and increasing to the right of 1, so there is (at least) a local minimum at $x = 1$. Since 1 is the only critical value, there is no way the graph of f can switch from increasing to decreasing (or decreasing to increasing) anywhere else. Therefore the value of f can never drop lower than $f(1)$ (*draw a picture!*), so there's an absolute minimum at $x = 1$.

23. If $f''(x) = x^5(x - 3)^4(x + 1)^3$, then $f(x)$ has exactly two inflection points.

A) True B) False

Inflection points may occur where f is defined and $f''(x) = 0$ or $f''(x)$ does not exist. Here, $f''(x)$ exists for all x, and $f''(x) = 0$ where $x = -1, 0, 3$. So there are at most three inflection points. A chart of where f'' is positive or negative shows that f'' switches sign (and f switches concavity) at $x = -1$ and $x = 0$ but not at $x = 3$. So there are exactly two inflection points for f.

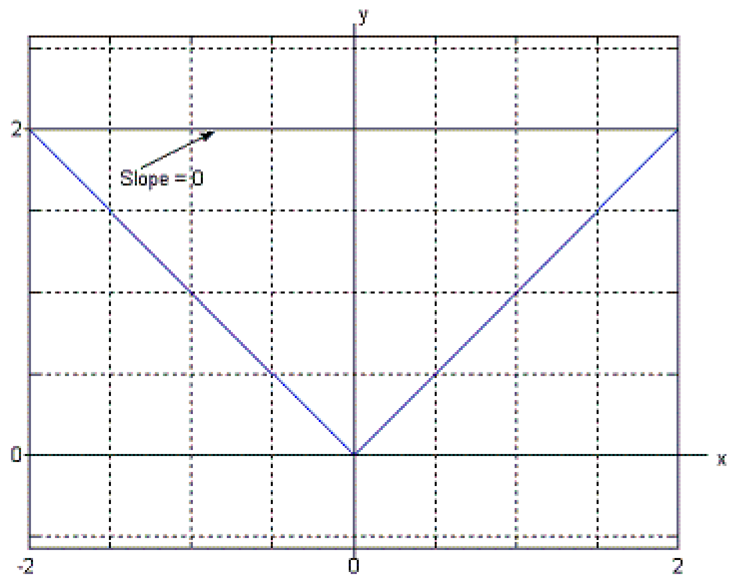
24. $\int_3^{x^2} \frac{e^t}{t}\, dt = \int_7^{x^2} \frac{e^t}{t}\, dt + C$, where C is a constant.

A) True B) False

Solution I: $\int_3^{x^2} \frac{e^t}{t}\, dt = \int_3^7 \frac{e^t}{t}\, dt + \int_7^{x^2} \frac{e^t}{t}\, dt$ and $\int_3^7 \frac{e^t}{t}\, dt$ is a constant.
Solution II: $\frac{d}{dx}\int_3^{x^2} \frac{e^t}{t}\, dt = \frac{e^{x^2}}{x^2}(2x) = \frac{d}{dx}\int_7^{x^2} \frac{e^t}{t}\, dt$. Since the functions have the same derivative for all x, they must differ by a constant.

25. Let $f(x) = |x|$. According to the Mean Value Theorem, there is a point c between -2 and 2 where $f'(c) = \frac{f(2) - f(-2)}{4}$.

A) True **B) False**



$\frac{f(2)-f(-2)}{4} = 0$ is the slope of the line segment form $(-2, 2)$ to $(2, 2)$ in the figure. There is no point c between -2 and 2 where $f'(c)$ ($=$ the slope of the tangent line to $y = |x|$) is 0. (The Mean Value Theorem doesn't apply because $|x|$ is not differentiable on this interval.)

26. $\frac{d}{ds}\int_1^2 s^2\, ds = 4$.

A) True **B) False**

$\int_1^2 s^2\, ds$ is a constant (in fact, $\int_1^2 s^2\, ds = \frac{7}{3}$), so $\frac{d}{ds}\int_1^2 s^2\, ds = 0$.

27. If $\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = 7$, then f must be continuous at 3.

A) True B) False

$\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = 7$ means that $f'(3) = 7$. Since f is differentiable at 3, it must also be continuous there.

28. $\lim_{x \to 0^+} x^{1/x}$ is an "indeterminate form" – that is, we must do some manipulation and perhaps use L'Hôpital's Rule to determine the value of the limit.

A) True **B) False**

As $x \to 0^+$, $\frac{1}{x} \to \infty$. For a number x between 0 and 1, raising it to power $\frac{1}{x} \to \infty$ gives smaller and smaller values. As $x \to 0^+$, the "x" and the "$\frac{1}{x}$" in $x^{1/x}$ are <u>cooperating</u> to cause $x^{1/x} \to 0$. "0^∞" is <u>not</u> indeterminate.