Derivatives and the Shapes of Curves

http://www.math.wustl.edu/~freiwald/Math131/shapes.pdf

$f(x)$	$f'(x)$	$f''(x)$
Increasing	$f'(x) > 0$	
Decreasing	$f'(x) < 0$	
Concave up	$f'(x)$ increasing	$f''(x) > 0$
Concave down	$f'(x)$ decreasing	$f''(x) < 0$

A critical point c of $f(x)$ is a point in the domain of f (not an endpoint) where either $f'(c) = 0$ or $f'(c)$ does not exist. Local maxima/minima, <u>if they exist</u>, can only occur at critical points (but some critical points may turn out to be neither a local maximum nor minimum).

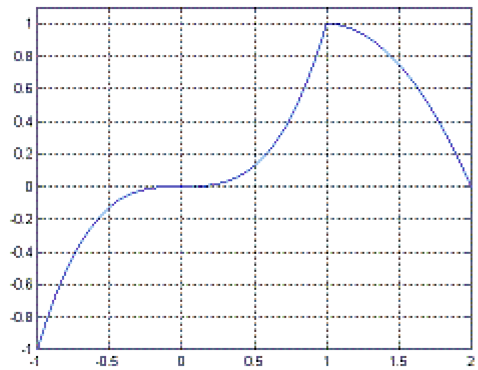
A <u>local</u> maximum occurs at a critical point c	if $f'(x)$ switches from positive to negative at c	$f''(c) < 0$ (assuming that near c, f'' is continuous)
A <u>local</u> minimum occurs at a critical point c	if $f'(x)$ switches from negative to positive at c	$f''(c) > 0$ (assuming that near c, f'' is continuous)

An inflection point occurs at a point c in the domain of f if the graph of $f(x)$ changes concavity (from up to down, or vice-versa) at the point c. Inflection points <u>may</u> occur at points where $f''(c) = 0$ or where $f''(c)$ doesn't exist (but some such points c may turn out not to be inflection points after all.

If c is a point where $f''(c) = 0$ or $f''(c)$ doesn't exist

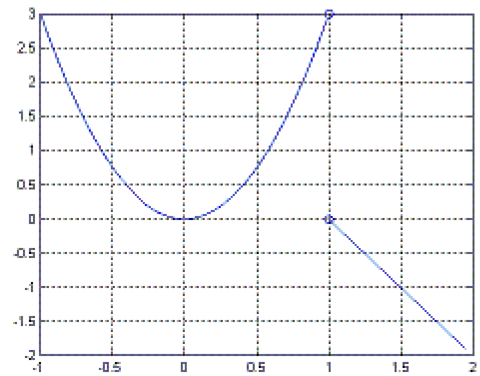
An inflection point occurs at c	if $f''(x)$ changes sign (positive to negative or vice-versa) at c.

$f(x)$



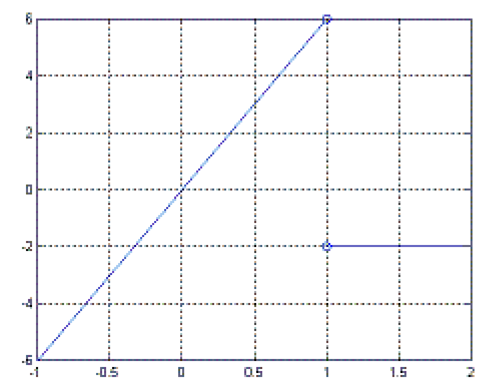
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1
1.5
2

$f'(x)$



3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
-0.5
0
0.5
1
1.5
2

$f''(x)$



6
4
2
0
-2
-4
-6
-1
-0.5
0
0.5
1
1.5
2