

Here is a slightly more realistic set of data involving the Leontief Open Economy Production Model. The data is from #13, p. 157 in the textbook.

The consumption matrix C is based on input-output data for the US economy in 1958, with data for 81 sectors grouped (for manageability here) into 7 larger sectors: (1) nonmetal household and personal products (2) final metal products (such as autos) (3) basic metal products and mining (4) basic nonmetal products and agriculture (5) energy (6) services and (7) entertainment and miscellaneous products. Units are in millions of dollars. (From Wassily W. Leontief, "The Structure of the US Economy", *Scientific American*, April 1965, pp. 30-32). Units are in millions of dollars.)

$$C = \begin{bmatrix} 0.1588 & 0.0064 & 0.0025 & 0.0304 & 0.0014 & 0.0083 & 0.1594 \\ 0.0057 & 0.2645 & 0.0436 & 0.0099 & 0.0083 & 0.0201 & 0.3413 \\ 0.0264 & 0.1506 & 0.3557 & 0.0139 & 0.0142 & 0.0070 & 0.0236 \\ 0.3299 & 0.0565 & 0.0495 & 0.3636 & 0.0204 & 0.0483 & 0.0649 \\ 0.0089 & 0.0081 & 0.0333 & 0.0295 & 0.3412 & 0.0237 & 0.0020 \\ 0.1190 & 0.0901 & 0.0996 & 0.1260 & 0.1722 & 0.2368 & 0.3369 \\ 0.0063 & 0.0126 & 0.0196 & 0.0098 & 0.0064 & 0.0132 & 0.0012 \end{bmatrix}$$

It turns out that $I - C$ is invertible, and for a demand vector $\mathbf{d} = \begin{bmatrix} 74000 \\ 56000 \\ 10500 \\ 25000 \\ 17500 \\ 196000 \\ 5000 \end{bmatrix}$

the solution to $\mathbf{x} = C\mathbf{x} + \mathbf{d}$ is easy with Matlab:

$$(I - C)^{-1} = \begin{bmatrix} 1.2212 & 0.0271 & 0.0226 & 0.0677 & 0.0135 & 0.0227 & 0.2167 \\ 0.0432 & 1.4046 & 0.1244 & 0.0466 & 0.0404 & 0.0516 & 0.5103 \\ 0.0806 & 0.3387 & 1.5927 & 0.0555 & 0.0508 & 0.0326 & 0.1810 \\ 0.6732 & 0.1905 & 0.1763 & 1.6448 & 0.0948 & 0.1266 & 0.3265 \\ 0.0636 & 0.0531 & 0.1010 & 0.0897 & 1.5393 & 0.0575 & 0.0590 \\ 0.3409 & 0.2711 & 0.2953 & 0.3253 & 0.3842 & 1.3674 & 0.6371 \\ 0.0213 & 0.0303 & 0.0392 & 0.0231 & 0.0175 & 0.0211 & 1.0246 \end{bmatrix}$$

(OVER)

and

$$\mathbf{x} = (I - C)^{-1}\mathbf{d} = \begin{bmatrix} 99580 \\ 97700 \\ 51230 \\ 13157 \\ 49490 \\ 32955 \\ 13840 \end{bmatrix}$$