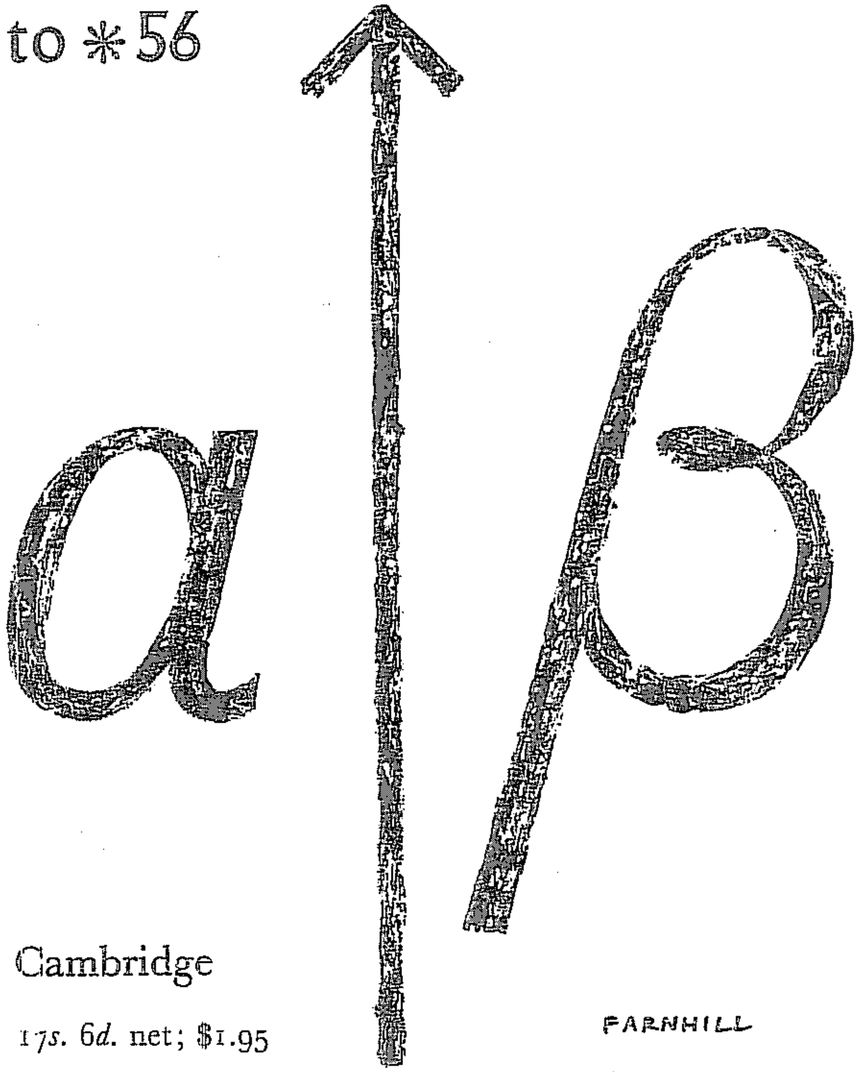
Principia Mathematica

to *56



Cambridge

17*s.* 6*d.* net; $1.95

FARNHILL

e cannot of course guarantee their
possible in the case of backward

s to previous writers have not very
sform whatever we have borrowed,
r notation. Our chief obligations
miliar with the literature of the
e as far as possible followed Peano,
y, by that of Frege or by that of
however, has had to be new, not
ymbolism of others, as through the
y symbolised. In all questions of
. Where we differ from him, it is
that he, in common with all other
some error to creep into his pre-
would have been almost impossible
e theory of series, our whole work
eometry we have had continually
Peano, Pieri, and Veblen.

A. N. W.
B. R.

CONTENTS

p. 345

$*52$. THE CARDINAL NUMBER 1

Summary of $*52$.

In this number, we introduce the cardinal number 1, defined as the class of all unit classes. The fact that 1 so defined is a cardinal number is not relevant at present, and cannot of course be proved until "cardinal number" has been defined. For the present, therefore, 1 is to be regarded simply as the class of all unit classes, unit classes being such classes as are of the form $\iota`x$ for some x.

Like Λ and V, 1 is ambiguous as to type: it means "all unit classes of the type in question." The symbol "$1(\alpha)$," where α is a type, will mean "all unit classes whose sole members belong to the type α" (cf. $*65$). Thus *e.g.* "$\xi \epsilon 1$ (Indiv)" will mean "ξ is a class consisting of one individual," if "Indiv" stands for the class of individuals.

The properties of 1 to be proved in the present number are what we may call *logical* as opposed to *arithmetical* properties, *i.e.* they are not concerned with the arithmetical operations (addition, etc.) which can be performed with 1, but with the relations of 1 to unit classes. The arithmetical properties of 1 will be considered later, in Part III.

The propositions of the present number which are most used are the following:

$*52{\cdot}16.\ \vdash :. \alpha \epsilon 1 . \equiv : \exists ! \alpha : x, y \epsilon \alpha . \supset_{x,y} . x = y$

I.e. α is a unit class if, and only if, it is not null, and all its members are identical.

$*52{\cdot}22.\ \vdash . \iota`x \epsilon 1$

$*52{\cdot}4.\ \vdash :. \alpha \epsilon 1 \cup \iota`\Lambda . \equiv : x, y \epsilon \alpha . \supset_{x,y} . x = y$

We shall define 0 as $\iota`\Lambda$. Thus the above proposition states that a class has one member or none when, and only when, all its members are identical.

$*52{\cdot}41.\ \vdash : \exists ! \alpha . \alpha \sim \epsilon 1 . \equiv . (\exists x, y) . x, y \epsilon \alpha . x \neq y$

This proposition is obtainable from $*52{\cdot}4$ by transposition, *i.e.* by negating each side of the equivalence.

$*52{\cdot}46.\ \vdash :. \alpha, \beta \epsilon 1 . \supset : \alpha \subset \beta . \equiv . \alpha = \beta . \equiv . \exists ! (\alpha \cap \beta)$

I.e. two unit classes are identical when, and only when, one is contained in the other, and when and only when they have a common part.

$*52{\cdot}01.\ 1 = \hat{\alpha}\{(\exists x) . \alpha = \iota`x\}$ Df

$*52{\cdot}1.\ \vdash : \alpha \epsilon 1 . \equiv . (\exists x) . \alpha = \iota`x$ [$*20{\cdot}3 . (*52{\cdot}01)$]

***52·11.** $\vdash :. \alpha \epsilon 1 . \equiv : (\exists x) : y \epsilon \alpha . \equiv_y . y = x$ [*52·1 . *51·14]

***52·12.** $\vdash : \hat{z}(\phi z) \epsilon 1 . \equiv . E ! (\imath x)(\phi x)$

Dem.

$\vdash . *52·11 . \supset \vdash :. \hat{z}(\phi z) \epsilon 1 . \equiv : (\exists x) : y \epsilon \hat{z}(\phi z) . \equiv_y . y = x :$

[*20·3] $\equiv : (\exists x) : \phi y . \equiv_y . y = x :$

[*14·11] $\equiv : E ! (\imath x)(\phi x) :. \supset \vdash . \text{Prop}$

***52·13.** $\vdash . 1 = D ‘ \iota$

Dem.

$\vdash . *51·131 . \supset \vdash : \alpha = \iota ‘ x . \equiv . \alpha \iota x :$

[*10·11·281] $\supset \vdash : (\exists x) . \alpha = \iota ‘ x . \equiv . (\exists x) . \alpha \iota x :$

[*52·1] $\supset \vdash : \alpha \epsilon 1 . \equiv . (\exists x) . \alpha \iota x$

[*33·13] $\equiv . \alpha \epsilon D ‘ \iota : \supset \vdash . \text{Prop}$

***52·14.** $\vdash . 1 = \iota ‘‘ V$ [*52·13 . *37·28]

***52·15.** $\vdash : \alpha \epsilon 1 . \equiv . E ! \breve{\iota} ‘ \alpha$ [*51·54 . *52·1]

***52·16.** $\vdash :. \alpha \epsilon 1 . \equiv : \exists ! \alpha : x, y \epsilon \alpha . \supset_{x,y} . x = y$ [*52·15 . *51·55 . *14·203]

***52·17.** $\vdash : \alpha \epsilon 1 . \equiv . \breve{\iota} ‘ \alpha = (\imath x)(x \epsilon \alpha)$ [*51·58 . *52·15]

***52·171.** $\vdash : \alpha \epsilon 1 . \equiv . E ! (\imath x)(x \epsilon \alpha)$ [*51·55 . *52·15]

***52·172.** $\vdash : \alpha \epsilon 1 . \equiv . \iota ‘ \breve{\iota} ‘ \alpha = \alpha$ [*51·52 . *52·15]

***52·173.** $\vdash : \alpha \epsilon 1 . \equiv . \breve{\iota} ‘ \alpha \epsilon \alpha$ [*51·53 . *52·15]

***52·18.** $\vdash :. \alpha \epsilon 1 . \equiv : (\exists x) : x \epsilon \alpha : y \epsilon \alpha . \supset_y . y = x$

Dem.

$\vdash . *51·141 . \supset \vdash :. (\exists x) . \alpha = \iota ‘ x . \equiv : (\exists x) : x \epsilon \alpha : y \epsilon \alpha . \supset_y . y = x$ (1)

$\vdash . (1) . *52·1 . \supset \vdash . \text{Prop}$

***52·181.** $\vdash :. \alpha \sim \epsilon 1 . \equiv : x \epsilon \alpha . \supset_x . (\exists y) . y \epsilon \alpha . y \neq x$ [*52·18 . *10·51]

***52·2.** $\vdash . 1 \subset \text{Cls}$

Dem.

$\vdash . *52·1 . \supset \vdash : \alpha \epsilon 1 . \supset . (\exists x) . \alpha = \iota ‘ x .$

[*51·11] $\supset . (\exists x) . \alpha = \hat{z}(z = x) .$

[*20·54] $\supset . (\exists x, \phi) . \hat{z}(\phi ! z) = \hat{z}(z = x) . \alpha = \hat{z}(\phi ! z) .$

[*10·5] $\supset . (\exists \phi) . \alpha = \hat{z}(\phi ! z) .$

[*20·4] $\supset . \alpha \epsilon \text{Cls} : \supset \vdash . \text{Prop}$

***52·21.** $\vdash . \Lambda \sim \epsilon 1$

Dem.

$\vdash . *52·16 . \supset \vdash : \alpha \epsilon 1 . \supset_\alpha . \exists ! \alpha :$

[*24·63] $\supset \vdash : \Lambda \sim \epsilon 1$

***52·22.** $\vdash . \iota ‘ x \epsilon 1$ [*51·12 . *14·28 . *10·24 . *52·1]