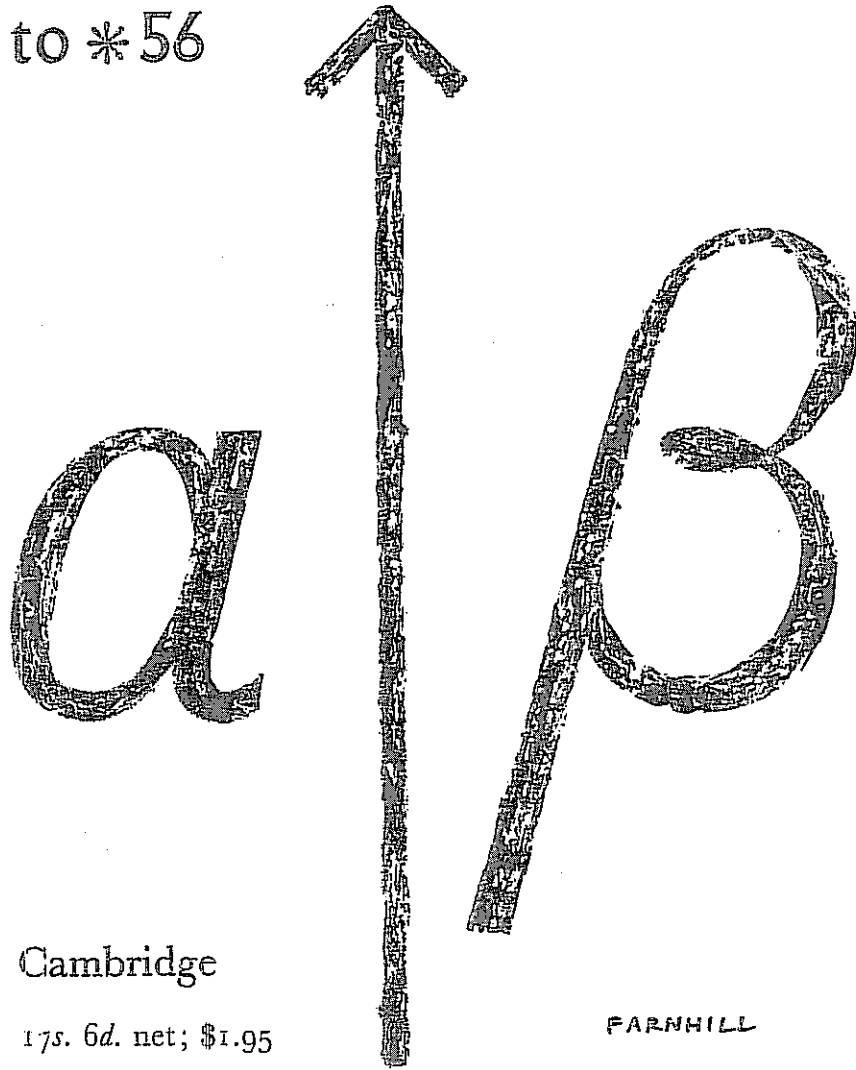


Principia Mathematica

to *56



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A. N. W.
B. R.

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#52. THE CARDINAL NUMBER 1

Summary of #52.

In this number, we introduce the cardinal number 1, defined as the class of all unit classes. The fact that 1 so defined is a cardinal number is not relevant at present, and cannot of course be proved until "cardinal number" has been defined. For the present, therefore, 1 is to be regarded simply as the class of all unit classes, unit classes being such classes as are of the form t'x for some x.

Like Δ and ∇ , 1 is ambiguous as to type: it means "all unit classes of the type in question." The symbol "1 (α)," where α is a type, will mean "all unit classes whose sole members belong to the type α " (cf. #65). Thus e.g. " $\xi \in 1$ (Indiv)" will mean " ξ is a class consisting of one individual," if "Indiv" stands for the class of individuals.

The properties of 1 to be proved in the present number are what we may call *logical* as opposed to *arithmetical* properties, i.e. they are not concerned with the arithmetical operations (addition, etc.) which can be performed with 1, but with the relations of 1 to unit classes. The arithmetical properties of 1 will be considered later, in Part III.

The propositions of the present number which are most used are the following:

#52-16. $\vdash : \alpha \in 1 \equiv : \nexists ! \alpha : x, y \in \alpha . \supset_{x,y} . x = y$

I.e. α is a unit class if, and only if, it is not null, and all its members are identical.

#52-22. $\vdash . t'x \in 1$

#52-4. $\vdash : \alpha \in 1 \cup t'\Delta \equiv : x, y \in \alpha . \supset_{x,y} . x = y$

We shall define 0 as $t'\Delta$. Thus the above proposition states that a class has one member or none when, and only when, all its members are identical.

#52-41. $\vdash : \nexists ! \alpha . \alpha \sim \epsilon 1 \equiv . (\nexists x, y) . x, y \in \alpha . x \neq y$

This proposition is obtainable from #52-4 by transposition, i.e. by negating each side of the equivalence.

#52-46. $\vdash : \alpha, \beta \in 1 . \supset : \alpha \subset \beta \equiv . \alpha = \beta \equiv . \nexists ! (\alpha \cap \beta)$

I.e. two unit classes are identical when, and only when, one is contained in the other, and when and only when they have a common part.

#52-01. $1 = \hat{\alpha} \{ (\nexists x) . \alpha = t'x \}$ Df

#52-1. $\vdash : \alpha \in 1 \equiv . (\nexists x) . \alpha = t'x$ [#20-3. (#52-01)]

#52.11. $\vdash: \alpha \in 1. \equiv: (\forall x): y \in \alpha. \equiv_y. y = \alpha$ [#52.1. #51.14]

#52.12. $\vdash: \hat{z}(\phi x) \in 1. \equiv. E!(\exists w)(\phi w)$

Dem.

\vdash . #52.11. $\supset \vdash: \hat{z}(\phi x) \in 1. \equiv: (\forall x): y \in \hat{z}(\phi x). \equiv_y. y = \alpha:$
 [#20.3] $\equiv: (\forall x): \phi y. \equiv_y. y = \alpha:$
 [#14.11] $\equiv: E!(\exists w)(\phi w): \supset \vdash$. Prop

#52.13. $\vdash. 1 = D't$

Dem.

\vdash . #51.131. $\supset \vdash: \alpha = t'x. \equiv. \alpha t x:$
 [#10.11.281] $\supset \vdash: (\forall x). \alpha = t'x. \equiv. (\forall x). \alpha t x:$
 [#52.1] $\supset \vdash: \alpha \in 1. \equiv. (\forall x). \alpha t x$
 [#33.13] $\equiv. \alpha \in D't: \supset \vdash$. Prop

#52.14. $\vdash. 1 = t''V$ [#52.13. #37.28]

#52.15. $\vdash: \alpha \in 1. \equiv. E! t'\alpha$ [#51.54. #52.1]

#52.16. $\vdash: \alpha \in 1. \equiv: \exists! \alpha: x, y \in \alpha. \supset_{x,y}. x = y$ [#52.15. #51.55. #14.203]

#52.17. $\vdash: \alpha \in 1. \equiv. t'\alpha = (\exists w)(w \in \alpha)$ [#51.58. #52.15]

#52.171. $\vdash: \alpha \in 1. \equiv. E!(\exists w)(w \in \alpha)$ [#51.55. #52.15]

#52.172. $\vdash: \alpha \in 1. \equiv. t' t'\alpha = \alpha$ [#51.52. #52.15]

#52.173. $\vdash: \alpha \in 1. \equiv. t'\alpha \in \alpha$ [#51.53. #52.15]

#52.18. $\vdash: \alpha \in 1. \equiv: (\forall x): x \in \alpha: y \in \alpha. \supset_y. y = \alpha$

Dem.

\vdash . #51.141. $\supset \vdash: (\forall x). \alpha = t'x. \equiv: (\forall x): x \in \alpha: y \in \alpha. \supset_y. y = \alpha$ (1)
 \vdash . (1). #52.1. $\supset \vdash$. Prop

#52.181. $\vdash: \alpha \sim \varepsilon 1. \equiv: x \in \alpha. \supset_x. (\forall y). y \in \alpha. y \neq x$ [#52.18. #10.51]

#52.2. $\vdash. 1 \subset Cls$

Dem.

\vdash . #52.1. $\supset \vdash: \alpha \in 1. \supset. (\forall x). \alpha = t'x.$
 [#51.11] $\supset. (\forall x). \alpha = \hat{z}(z = x).$
 [#20.54] $\supset. (\forall x, \phi). \hat{z}(\phi! z) = \hat{z}(z = x). \alpha = \hat{z}(\phi! z).$
 [#10.5] $\supset. (\forall \phi). \alpha = \hat{z}(\phi! z).$
 [#20.4] $\supset. \alpha \in Cls: \supset \vdash$. Prop

#52.21. $\vdash. \Lambda \sim \varepsilon 1$

Dem.

\vdash . #52.16. $\supset \vdash: \alpha \in 1. \supset_x. \exists! \alpha:$
 [#24.63] $\supset \vdash: \Lambda \sim \varepsilon 1$

#52.22. $\vdash. t'x \in 1$ [#51.12. #14.28. #10.24. #52.1]