

As mentioned in the lecture, saying

“two propositional forms \*\*\*\* and \*\*\*\*\* are equivalent” is the same as saying

“ \*\*\*\*  $\Leftrightarrow$  \*\*\*\*\* is a tautology ”

Some useful tautologies

$$\begin{array}{lll} P \vee (Q \vee R) & \Leftrightarrow & (P \vee Q) \vee R \quad (\text{and also, for } \wedge) \\ P \vee (Q \wedge R) & \Leftrightarrow & (P \vee Q) \wedge (P \vee R) \\ P \wedge (Q \vee R) & \Leftrightarrow & (P \wedge Q) \vee (P \wedge R) \end{array}$$

**Excluded Middle**  $P \vee \sim P$

**DeMorgan's Rules**

$$\begin{array}{lll} \sim (P \vee Q) & \Leftrightarrow & \sim P \wedge \sim Q \\ \sim (P \wedge Q) & \Leftrightarrow & \sim P \vee \sim Q \end{array}$$

**Modus Ponens**  $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

$$(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$$

(a conditional is equivalent to its contrapositive)

$$(P \Leftrightarrow Q) \text{ is equivalent to } (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

$$(P \Rightarrow Q) \text{ is equivalent to } Q \vee \sim P$$

$$(P \Rightarrow Q) \text{ is equivalent to } \sim (P \wedge \sim Q)$$