The discussion in the first half of the first 310 lecture was by design somewhat rambling. I thought it might be good to summarize some of the points that came up. I think these ideas make a useful "context" for the course. We'll be talking off-and-on about them, and other general ideas, throughout the course.

What makes mathematics different from other fields? Some ideas:

1. There is a commonly agreed on foundation for mathematics: set theory. Set theory can studied in great detail and very rigorously. We will take a look at the foundation starting in chapter 2.

2. There is available a very formal "language" that mathematicians can use to make meanings perfectly clear. In everyday practice (the sort of stuff you see throughout textbooks, say) mathematicians write a fairly careful mix of ordinary English and formal language. But whenever there's disagreement about what something means, mathematicians can rewrite things in more and more formal language until no ambiguity remains.

In particular, this means that the "objects" being talked about in mathematics can be very precisely defined. Every mathematician knows exactly what every other mathematician means by, say, "a differentiable function." (*In contrast, can philosophers agree about what "a good" is ?*)

3. There is an established methodology for getting to new results from what is known (starting with the foundation, set theory): proofs, which are based on the rules of logic. Proofs can also be made very formal when necessary, but in practice mathematicians usually write proofs in a careful but more casual style, which can be made more and more precise if there's a subtle point, confusion or disagreement.

With these distinctives, mathematicians rarely have serious arguments about actual mathematics. Of course, as in any human enterprise, they can argue about other things that have to do with "style," "taste," and "philosophy."

What *can* mathematicians argue about? Some illustrations:

1. *Whether a proof is correct* These are usually temporary, polite disagreements that can be resolved, with some patience, by looking carefully at steps in a proof.

2. *Whether a proof is a "good" proof* Often there's more than one way to prove a result. Some proofs may be short but make use of deep and complicated tools; another proof of the same result might only use more elementary tools and be longer – but it might also give more intuitive insight into "why" the result is true. This issue is a value judgment – not to be confused with correctness of the mathematics itself.

3. *Whether a piece of mathematics is worthwhile* Does it fit into an "active" area of mathematical research? Does it lead one to suspect that even more is true? Does it raise new interesting questions? Does it cast light on other parts of mathematics (or, for that matter, applications). This is also a value judgment.

4. *Issues related to the "philosophy of mathematics"* For example, what is a number (e.g. "2"). Do numbers have some ideal existence "outside" of us (for example, as "ideas" in the mind of God) and we somehow manage to "see" some partial information about them and then deduce new conclusions? Or are numbers simply symbols that we invent and manipulate according to certain rules. Is the idea of "2" "hardwired" into human brains? How about "18281828459045" ? Are mathematicians "discovering things" or "inventing things"? These are all perfectly reasonable questions and can cause heated discussion among those interested in them.

But these are really questions about philosophy. Mathematicians can have their seminars, do research that everyone else agrees with, and teach their classes without any of these issues ever needing to come up. The reason is that there is agreement on a foundation (set theory) and numbers (like 2) can be precisely defined in terms of sets. Even if a mathematician doesn't like the way the philosophy or details of the foundation have been handled, they can still agree to work with "2" as long it has been precisely defined and behaves in the "correct" way . All that matters is that we agree on how numbers <u>behave</u>.. If we like, we can disagree after work over a beer about what numbers <u>really are</u>. For doing mathematics, it doesn't matter what they "really are."

5. *Teaching methods* But these are really issues of "what works" for a given teacher and class, and an area of study for students in education and psychology. They aren't mathematical issues.

5. *All sorts of personal (and personality) issues* For example, national or university politics. There were bitter exchanges in the mathematical community over the Vietnam war but these were not mathematical issues. (*Could historians say the same?*) Someone has remarked that the reason university politics sometimes gets so bitter is because there's so little at stake.

The "moral" I'm pushing here is that mathematics has a very settled foundation and methodology at the core. As far as mathematics <u>itself</u> is concerned, it's a very "peaceable" field of study. Disagreements come up, but they are usually about (related) nonmathematical issues.