Homework 1: Due 09/07/2017

1. Let $-\infty < a < b < \infty$ be real numbers. Consider

$$\mathcal{C}_1 = \{(a, b]\}, \mathcal{C}_2 = \{[a, b]\}, \mathcal{C}_3 = \{(-\infty, a)\}, \mathcal{C}_4 = \{(-\infty, a]\},$$

be the collections of different type of intervals. Show that $\mathcal{B} = \sigma(\mathcal{C}_i), i = 1, 2, 3, 4,$ where $\mathcal{B}$ is the Borel σ-field on $\mathcal{R}$.

2. Let $(\Omega, \mathcal{F})$ be measurable space, and μ be a non-negative set function on it with $\mu(\varnothing) = 0$. Show that μ statistfies (i) finite additivity, i.e.

$$\mu(A \cup B) = \mu(A) + \mu(B), \forall \text{ disjoint } A, B \in \mathcal{F},$$

and (ii) countinuity from below, i.e.

$$\mu(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} \mu(A_n), \text{ for } A_1 \subset A_2 \subset \cdots, A_n \in \mathcal{F},$$

if and only if it satisfies sigma additivity.

3. π- and λ- system.
(1) Shao (2003), P74, Ex. 5.
(2) Shao (2003), P74, Ex. 12.

4. Shao (2003), P74, Ex. 9.
Let P be a probability measure on $(\mathbb{R}, \mathcal{B})$. Define

$$F(x) = P((-\infty, x]), \forall x \in \mathbb{R}$$

be c.d.f. of P.
(1) Prove properties of c.d.f.
(2) Show there exists a unique measure for a given c.d.f.

5. Shao (2003), P74, Ex. 14.
Prove properties of measurable function.