Homework 11: Due 12/7/2017

1. Let $X_1, \cdots, X_n \stackrel{\text{iid}}{\sim} N(\mu, \sigma^2), \mu \in \mathbb{R}$, where σ^2 is known. Consider estimation problem of $\theta = \mu$. Let $T_{1n} = \bar{X}_n$. In Bayesian framework, the parameter μ is random instead of fixed. Assume $\mu \sim N(\mu_0, \sigma_0^2)$ with known μ_0 and σ_0^2.

   (a) Derive the estimator $T_{2n} = E(\mu | X_1, \cdots, X_n)$.

   (b) The average risk of an estimator, T, with respect to squared error loss is

   $$r(T, \Pi) = \int R(\theta, T) d\Pi = \int E[(\theta - T(X))^2 | \theta] d\Pi (= E[(\theta - T(X))^2]).$$

   Show that when Π is $N(\mu_0, \sigma_0)$,

   $$T_{2n} = \arg\min_T r(T, \Pi).$$

   Here T_{2n} is called a Bayesian estimator (rule).

   (c) Calculate exact MSE of both estimators. Can you conclude that one estimator is better than the other in terms of the MSE?

   (d) Find the asymptotic relative efficiency of T_{1n} w.r.t. T_{2n}.

2. Problem 119 on page 159 of Shao (2003).

3. Problem 117 on page 159 + Problem 100 on page 228 of Shao (2003).

4. Asymptotic bias and asymptotic MSE do not always consistent with bias and MSE in the finite sample.
   Problem 115 on page 158 of Shao (2003).