Homework 2: Due 09/14/2017

1. Fubini's Theorem is only valid when the function is integrable or nonnegative. Find a counter example of Fubini's Theorem when the condition is not satisfied.

2. Shao (2003), P74, Ex. 31.
   Prove integration by parts. Let F and G be continuous c.d.f. on $\mathbb{R}$, show that

   $$\int_{(a,b]} G(x)dF(x) = F(b)G(b) - F(a)G(a) - \int_{(a,b]} F(x)dG(x).$$

   Remark: The result can be generalized to allow some discontinuity points as long as they are not common for F and G.

3. Shao (2003), P74, Ex. 7, 23, 35.
   Let $\mu = \sum_{n=1}^{\infty} a_n \mu_n$, where μ_n is a sequence of measures on a common measurable space, and $\{a_n\}$, be a sequence of positive numbers.

   (a) Show that μ is a measure.

   (b) For an integrable function f w.r.t. μ_1 and μ_2, show that

   $$\int fd(\mu_1 + \mu_2) = \int fd\mu_1 + \int fd\mu_2$$

   (c) Consider probability space. Let μ_n be probability measures and $\sum a_n = 1$. Then μ is also a probability measure. Show that $\mu_n \ll \nu, \forall n$ iff $\mu \ll \nu$. Furthermore, the density

   $$\frac{d\mu}{d\nu} = \sum_{n=1}^{\infty} a_n \frac{d\mu_n}{d\nu}$$

4. Shao (2003), P74, Ex. 36.
   Let F_i be a c.d.f. having a Lebesgue p.d.f. f_i, $i = 1, 2$. Assume that there is a $c \in \mathcal{R}$ such that $F_1(c) < F_2(c)$. Define

   $$F(x) = \begin{cases} F_1(x), & -\infty < x < c \\ F_2(x), & c \leq x < \infty. \end{cases}$$

   Show that the probability measure P corresponding to F satisfies $P \ll m + \delta_c$ and find $dP/d(m + \delta_c)$, where $m + \delta_c$ is given in (1.23).