Homework 8: Due 11/09/2017

1. Problem 22 on page 144 of Shao (2003).

2. Problem 23 on page 144 of Shao (2003).

3. Which of the following familis of distributions are exponential families? (Prove or disprove.)

   (a) Uniform distribution: $U(0, \theta)$.

   (b) Discrete uniform distribution: $f(x, \theta) = 1/10, x = \theta, \theta + 0.1, \cdots, \theta + 0.9$.

   (c) $N(\theta, \theta^2)$, $\theta > 0$.

   (d) $f(x, \theta) = \frac{2(x+\theta)}{1+2\theta}, 0 < x < 1, \theta > 0$.

   (e) $X \sim Bin(\theta, n)$, the conditional distribution of X given $X > 0$.

   (f) Student's t-distribution.

   (g) Double exponential distribution: $DE(\mu, \theta), \mu \in \mathbb{R}, \theta > 0$.

   (h) Double exponential distribution with known mean: $DE(\mu_0, \theta), \theta > 0$, μ_0 is a known constant.

   (i) **Power series** distribution: $f(x, \theta) = \frac{\gamma(x)\theta^x}{\sum_{x=0}^{\infty} \gamma(x)\theta^x}, x = 0, 1, 2, \cdots, \gamma(x) \geq 0, \theta > 0$

4. Problem 3 +(g) on page 143 of Shao (2003).
   (g)Let P_θ be the distribution of $Y = (Y_1, \cdots, Y_n)^T$ given $x = (x_1, \cdots, x_n)^T$ in a linear regression model

   $$Y_i = \beta_0 + \beta_1 x_i + e_i, \quad i = 1, 2, \cdots, n$$

   where $e_i \overset{\text{iid}}{\sim} N(0, \sigma^2)$. Here $\theta = (\beta_1, \beta_2, \sigma^2)^T \in \Theta = \mathbb{R}^2 \times (0, \infty)$.

5. Problem 14 + 16 on page 144 of Shao (2003).