MATH 233 HOMEWORK 5 FALL 2004

For all MATLAB graphs, include a title with your name in it (printed by MATLAB) and label the axes.

(1) (§10.5, #32 p.742 and ML §7.4.3 p.128)
  (a) Find a parametric representation for the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(b, 0, 0)$ and radius $a < b$. Write by hand the parametric equations in the top margin of the graph you print out in the next part.
  (b) Use MATLAB and the equations you have obtained to plot the torus for the case $a = 1$ and $b = 3$. Set *axis equal* to get an undistorted figure.

(2) (See #2 p. 134 of ML and #4 p. 765, §11.2) Consider the function

$$f(x, y) = \frac{4xy}{x^2 + 3y^2}$$

  (a) Plot the graph of this function over the domain $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Experiment with the view to see which best shows the features of the graph at $(0, 0)$. Use a fine mesh, say at increments of .02, to get adequate detail.
  (b) Write the following work in the space above and below your printed graph.
    (i) Show that restricted to any line through the origin, this function is constant. For example, do this by using polar coordinates to see that this function restricted to such a line depends only on θ. How does this agree with the graph?
    (ii) From this information, what can you conclude about $\lim_{(x,y)\to(0,0)} f(x, y)$?

(3) (#36 p.777 of §11.3) Consider the function $f(x, y) = \sin(2x + 3y)$. Use MATLAB to plot the graph of $z = f(-6, y)$ over $2 \leq y \leq 6$. (That's a curve in the yz-plane. Label the axes correctly). On the print out use a straight edge to draw the tangent line to the graph at $y = 4$. In the top and bottom margins find $f_y(-6, 4)$ and explain what this has to do with the slope of the line you have drawn on the graph.

(4) (#8 p.788 §11.4 and ML §8.3.1 pp. 136-137). Consider the function

$$f(x, y) = \frac{\sqrt{1 + 4x^2 + 4y^2}}{1 + x^4 + y^4}$$

  (a) Find the linearization, $L(x, y) = f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1)$.
  (b) Use MATLAB to plot on the same figure the graph of $z = f(x, y)$ over $[-2, 2] \times [-2, 2]$ and the graph of $z = L(x, y)$ over the domain $[0, 2] \times [0, 2]$.