

Solutions to HWK#1

- 1) The closest points on the  $xy$ ,  $yz$  &  $xz$  planes, respectively, to the point  $(3, 7, -5)$  are the points  $(3, 7, 0)$ ,  $(0, 7, -5)$  &  $(3, 0, -5)$ , respectively. Hence we get a) 5, b) 3 and c) 7. Similarly we can find the closest points to  $(3, 7, -5)$  on the  $x$ -axis,  $y$ -axis &  $z$ -axis, respectively. They are  $(3, 0, 0)$ ,  $(0, 7, 0)$  &  $(0, 0, -5)$ . We then get that d)  $\sqrt{49 + 25} = \sqrt{74}$ , e)  $\sqrt{9 + 25} = \sqrt{34}$ , f)  $\sqrt{9 + 49} = \sqrt{58}$ .

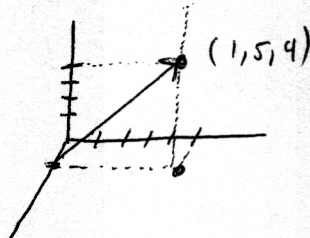
- 2) We can divide by 3 and get  $x^2 + y^2 + \frac{2}{3}y + z^2 - \frac{2}{3}z = 3$ . This can be rewritten as  $x^2 + (y + \frac{1}{3})^2 + (z - \frac{1}{3})^2 = 3 + \frac{1}{9} + \frac{1}{9} = \frac{29}{9}$ . Then center is  $(0, -\frac{1}{3}, \frac{1}{3})$  and radius is  $\frac{\sqrt{29}}{3}$ .

- 3) A plane perpendicular to the  $z$ -axis just means it is a plane parallel to the  $xy$ -plane or has formula  $z = c$ . Since  $(1, 1, 3)$  is on this plane, the plane must be  $z = 3$ . The sphere in question is  $x^2 + y^2 + z^2 = 25$ . The intersection of the plane and the sphere gives  $x^2 + y^2 + 9 = 25$  or  $x^2 + y^2 = 16$ . Conclusion: We have circle of radius 4 with center  $(0, 0, 3)$  lying on plane  $z = 3$ .

$$\text{or } \left\{ (x, y, z) / x^2 + y^2 = 16 \text{ and } z = 3 \right\}.$$

- 4) a)  $\vec{AB} = \langle 2 - 1, 3 - (-2), 4 - 0 \rangle = \langle 1, 5, 4 \rangle$ .

b)



- 5) Given  $\mathbf{a} = \langle 3, 0, -2 \rangle$  and  $\mathbf{b} = \langle 1, -1, 1 \rangle$  then:

$$|\mathbf{a}| = \sqrt{13}, \quad \mathbf{a} + \mathbf{b} = \langle 4, -1, -1 \rangle, \quad \mathbf{a} - \mathbf{b} = \langle 2, 1, -3 \rangle, \quad 2\mathbf{a} = \langle 6, 0, -4 \rangle, \quad 3\mathbf{a} + 2\mathbf{b} = \langle 11, -2, -4 \rangle.$$

- 6)  $|\mathbf{a}| = \sqrt{81 + 4 + 36} = 11$  and  $\mathbf{a} = 11 \left( \frac{9}{11} \mathbf{i} - \frac{2}{11} \mathbf{j} + \frac{6}{11} \mathbf{k} \right)$ .

Note:  $\langle \frac{9}{11}, -\frac{2}{11}, \frac{6}{11} \rangle$  is unit vector in the direction of  $\mathbf{a}$ .