Math 233 FALL 2004

Solutions to HWK#2

1) We have $\cos(\theta) = \frac{a \cdot b}{|a|\,|b|} = \frac{5}{49}$. Then $\theta = \arccos(\frac{5}{49}) \sim 1.33\ radians$.

2) a) We are given that $\frac{b \cdot a}{|a|} = \frac{a \cdot b}{|b|}$. Since the numerators are the same we arrive at the conclusion that either $|\mathbf{a}| = |\mathbf{b}|$ or $\mathbf{a} \cdot \mathbf{b} = 0$ (i.e. they are orthogonal).

b) Since $\text{comp}_a(\mathbf{b})$ is the **magnitude** of $\text{proj}_a(\mathbf{b})$ from part (a) we already know that either $|\mathbf{a}| = |\mathbf{b}|$ or they are orthogonal . The equation for proj gives us $(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2})\,\mathbf{a} = (\frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{b}|^2})\,\mathbf{b}$. Therefore either $\mathbf{a} = \mathbf{b}$ or they are orthogoal .

3) The set of all vectors parallel to $<1,1,0>$ must be of the form $<a,a,0>$ for some real number a . Next, all vectors parallel to $<1,1,0>$ must be of the form $<b, -b, c>$ for any numbers b and c . We are trying to find values of a , b and c so that $<0,3,4> = <a,a,0> + <b,-b,c> = <a+b, a-b, c>$. Hence $a+b=0$ and $b = -a$. $a - b = 3$ means $a = \frac{3}{2}$ and $b = -\frac{3}{2}$. Finally $c = 4$. Conclusion $<0,3,4> = <\frac{3}{2}, \frac{3}{2}, 0> + <-\frac{3}{2}, \frac{3}{2}, 4>$.

4) **a x b** is orthogonal to **a** & **b** . Calculation yields **a x b** $= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} =$ $<1, -1, -2>$. Since $|\mathbf{a \times b}| = \sqrt{6}$, we get that the the two unit vectors are $\pm \frac{1}{\sqrt{6}} < 1, -1, -2 >$.

5) a) We need a vector orthogonal to **PQ** and **PR**. One possibility is **PQ x PR**, which is $\begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ 3 & 2 & -1 \end{vmatrix} = <5, -8, -1>$.

b) Area of triangle PQR is half the area of the parallelogram , which is $\frac{1}{2}\,|\mathbf{PQ \times PR}| = \frac{3}{2}\sqrt{10}$.

6) We know that the volume of the parallelepiped can be gotten from $|\mathbf{a} \cdot (\mathbf{b \times c})|$.

This can be computed from :

$$\text{Abs}\left(\begin{vmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \right) = |-19| = \mathbf{19} .$$