Homework 10, Math 4121, due 10, April 2014

(1) Prove that $\frac{x \log x}{1+x^2} \in L^1([0,1])$.

(2) Prove that $\frac{1}{1+x^4 \sin^2 x} \in L^1([0,\infty))$.

(3) Let $f : [0,1] \to \mathbb{R}$ be continuous, $f(0) = 0$ and $f'(0)$ exists. Prove that $f(x)x^r \in L^1([0,1])$ if $r > -2$.

(4) Calculate $\Gamma(n+\frac{1}{2})$ for non-negative integers n. (Hint: $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.)

(5) If $\phi : (a,b) \to \mathbb{R}$ is continuous, prove that it is convex if and only if $\phi(\frac{x+y}{2}) \leq \frac{1}{2}(\phi(x) + \phi(y))$.

(6) Let $\phi : (a,b) \to (0,\infty)$ be any function such that $\log \phi$ is convex. Prove that ϕ is convex.