Homework 11, Math 308, due April 26th

(1) Write down series solutions for the following differential equations.
   (a) \( y' - y = f(x) \) where \( f(x) = \sum_{n=0}^{\infty} a_n x^n \) with initial condition \( y(0) = 0 \).
   (b) \( x^2 y'' + xy' + y = 0 \).

(2) Calculate \( P_3(x) \) and \( P_4(x) \), the third and fourth Legendre polynomials.

(3) If \( F(x), A(x) \) are polynomials and \( 0 \leq k \leq n \) are integers, show that we can write
   \[ \frac{d^k F^n(x)}{dx^k} A(x) \] as \( F^{n-k}(x)G(x) \) for a polynomial \( G(x) \).

(4) Find the Legendre series for the function \( f(x) = 0, -1 < x < 0 \) and \( f(x) = x, 0 < x < 1 \).

(5) Find the Legendre series for \( f(x) = P_n'(x) \).

(6) Let \( \Phi(x, h) = \sum P_n(x)h^n \) be the generating function for Legendre Polynomials. Show that \( (x - h)\frac{\partial \Phi}{\partial x} = h\frac{\partial \Phi}{\partial h} \).