Homework 11, Math 308, due April 26th

(1) Write down series solutions for the following diferential equations.
   (a) $y' - y = f(x)$ where $f(x) = \sum_{n=0}^{\infty} a_n x^n$ with initial condition $y(0) = 0$.
   (b) $x^2 y'' + xy' + y = 0$.
(2) Calculate $P_3(x)$ and $P_4(x)$, the third and fourth Legendre polynomials.
(3) If $F(x), A(x)$ are polynomials and $0 \leq k \leq n$ are integers, show that we can write $\frac{d^k F^n(x) A(x)}{dx^k}$ as $F^{n-k}(x)G(x)$ for a polynomial $G(x)$.
(4) Find the Legendre series for the function $f(x) = 0, -1 < x < 0$ and $f(x) = x, 0 < x < 1$.
(5) Find the Legendre series for $f(x) = P_n'(x)$.
(6) Let $\Phi(x, h) = \sum P_n(x) h^n$ be the generating function for Legendre Polynomials. Show that $(x - h)\frac{\partial \Phi}{\partial x} = h\frac{\partial \Phi}{\partial h}$.