Homework 12, Math 308, not to be collected

(1) Solve the Laplace equation $\nabla^2 u(x,y) = 0$ with the boundary conditions $u(x,0) = 0$ and $u(0,y) = \sin y$ in the form $u(x,y) = f(x)g(y)$.

(2) Solve the Laplace equation $\nabla^2 u(x,t) = 0$ in Fourier series given the boundary conditions $0 \leq x \leq 20$ with $u(0,t) = u(20,t) = 0$, $u(x,t) \leftarrow 0$ as $t \leftarrow \infty$ and $u(x,0) = 0, x < 10, = 100, x > 10$.

(3) Solve the diffusion equation $\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u(x,t)}{\partial t}$, for a constant α with the boundary conditions, $0 \leq x \leq 10$, $u(0,t) = u(10,t) = 0$ and $u(x,0) = 0, x < 5$ and $u(x,0) = 20, x > 5$.

(4) A bar of length 2 is initially at 0^o. From $t = 0$, the end $x = 0$ is held at 0^o and the end $x = 2$ is held at 100^o. Find the time-dependant temperature distribution.