Homework 2, Math 308, Spring 2010, due Feb 15th

(1) Write the following complex numbers in the form $re^{i\theta}$.
   (a) i^3
   (b) $\frac{3+i}{2+i}$
   (c) $(i+\sqrt{3})^2$

(2) Complex numbers have many of the properties of real numbers, but it lacks one important property. You can not say a complex number is less than another. This is meaningless. Show that if z_1, z_2 are complex numbers, $|z_1 + z_2| \leq |z_1| + |z_2|$ (Triangle inequality).

(3) If $a \neq 0$ is a complex number, find all complex numbers z satisfying the equation $e^z = a$.

(4) Find all complex numbers z satisfying the equation $z^8 = 1$.

(5) Show that for any natural number n, $z^n = 1$ has n distinct complex solutions. If $\omega_0, \ldots, \omega_{n-1}$ are these solutions, show that $\sum_{i=0}^{n-1} \omega_i = 0$.

(6) Calculate $\sum_{n=0}^{\infty} \frac{(2+i\pi)^n}{2^n n!}$.

(7) Find the determinants of the following matrices.

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

(8) Calculate $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$ for the following vectors $\mathbf{A}, \mathbf{B}$.
   (a) $\mathbf{A} = 2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - \mathbf{j}$.
   (b) $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{A}$.

(9) Let $\mathbf{A}, \mathbf{B}$ be vectors in 3-space, with $\mathbf{A} \times \mathbf{B} \neq 0$.
   (a) Show that the determinant d given by,

$$\begin{vmatrix} \mathbf{A} \cdot \mathbf{A} & \mathbf{A} \cdot \mathbf{B} \\ \mathbf{A} \cdot \mathbf{B} & \mathbf{B} \cdot \mathbf{B} \end{vmatrix}$$

   is not zero.
   (b) If $\mathbf{C}$ is any other vector, show that there exists real numbers a, b, c so that $\mathbf{C} = a\mathbf{A} + b\mathbf{B} + c\mathbf{A} \times \mathbf{B}$.

(10) Calculate the distance from the point $P = (4, 7, 0)$ to the line given parametrically by $x = 2t, y = 3t, z = 4t$.