Homework 3, Math 308, Spring 2010, due Feb 22nd

(1) For For $u = e^x \cos y$ verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ and $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

(2) For a function $f(x, y)$ in two variables let the Maclaurin series be $f(x, y) = \sum_{i,j=0}^{\infty} a_{ij} x^i y^j$. Find expressions for a_{ij} in terms of i, j for the following functions.
  (a) $f(x, y) = \sin(x + y)$
  (b) $f(x, y) = \frac{1}{1-xy}$

(3) The thin lens formula is $i^{-1} + o^{-1} = f^{-1}$, where f is the focal length, o and i, the distances from the lens to the object and image respectively. If $i = 15$ when $o = 10$, use differentials to find i when $o = 10.1$, for a given lens.

(4) Use differentials to estimate the change in

$$f(x) = \int_0^x \frac{e^{-t}}{t^2 + 0.51} dt$$

if x changes from 0.7 to 0.71.

(5) Given that $z = (x + y)^5$, and $y = \sin 10x$, find $\frac{dz}{dx}$.

(6) If P_i's are finitely many points in the plane with masses m_i at these points, the formula for the moment of inertia at a point P is given by $\sum m_i d(P_i, P)^2$, where $d(P_i, P)$ is the distance from P_i to P. Given masses m_i at $P_i = (x_i, y_i)$ find the co-ordinates of the point P where the moment of inertia is least.

(7) If we made a can with a circular base and top whose total (outer) surface area is 24π square meters, what would be the maximum possible volume?