Homework 3, Math 308, Spring 2010, due Feb 22nd

(1) For \( u = e^x \cos y \) verify that \( \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \) and \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \).

(2) For a function \( f(x, y) \) in two variables let the Maclaurin series be \( f(x, y) = \sum_{i,j=0}^{\infty} a_{ij} x^i y^j \). Find expressions for \( a_{ij} \) in terms of \( i, j \) for the following functions.
   (a) \( f(x, y) = \sin(x + y) \)
   (b) \( f(x, y) = \frac{1}{1-xy} \)

(3) The thin lens formula is \( i^{-1} + o^{-1} = f^{-1} \), where \( f \) is the focal length, \( o \) and \( i \), the distances from the lens to the object and image respectively. If \( i = 15 \) when \( o = 10 \), use differentials to find \( i \) when \( o = 10.1 \), for a given lens.

(4) Use differentials to estimate the change in \( f(x) = \int_0^x e^{-t} dt \) if \( x \) changes from 0.7 to 0.71.

(5) Given that \( z = (x + y)^5 \), and \( y = \sin 10x \), find \( \frac{dz}{dx} \).

(6) If \( P_i \)'s are finitely many points in the plane with masses \( m_i \) at these points, the formula for the moment of inertia at a point \( P \) is given by \( \sum m_i d(P_i, P)^2 \), where \( d(P_i, P) \) is the distance from \( P_i \) to \( P \). Given masses \( m_i \) at \( P_i = (x_i, y_i) \) find the co-ordinates of the point \( P \) where the moment of inertia is least.

(7) If we made a can with a circular base and top whose total (outer) surface area is \( 24\pi \) square meters, what would be the maximum possible volume?