

Homework 4, Math 308, Spring 2010, due March 1st

- (1) Given $x^2 + y^2 = 2st - 10$ and $2xy = s^2 - t^2$, find $\frac{\partial x}{\partial s}$, $\frac{\partial x}{\partial t}$, $\frac{\partial y}{\partial s}$ and $\frac{\partial y}{\partial t}$ at $(x, y, s, t) = (4, 2, 5, 3)$.
- (2) Given $f(x, y, z) = 0$ and $g(x, y, z) = 0$, find a formula for $\frac{dy}{dx}$.
- (3) Solve for z as a function of x, y , given,

$$2\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 10\frac{\partial^2 z}{\partial y^2} = 0.$$

- (4) Express z as a function of x, y , given,

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 1.$$

- (5) Calculate dy/dx if $y = \int_{x^2}^{x^3} \sin t dt$.
- (6) Calculate,

$$\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x \frac{1 - \cos t}{t^2} dt.$$

- (7) Given $\int_0^\infty e^{-at^2} dt = \frac{1}{2}\sqrt{\pi/a}$, calculate $\int_0^\infty t^2 e^{-at^2} dt$.
- (8) Show that $\frac{d}{dx} \int_{\cos x}^{\sin x} \sqrt{1 - t^2} dt = 1$.