Homework 7, Math 308, due March 29th

(1) Compute the diveregence and curl of the vector field $\mathbf{V} = x\sin y\mathbf{i} + \cos y\mathbf{j} + xy\mathbf{k}$.

(2) Calculate the Laplacian $\nabla^2\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$.

(3) Calculate the line integral $\int \frac{xdy-ydx}{x^2+y^2}$ along the following path. Start from $(1,0)$, go along the x-axis to $(a,0)$ where $a>0$, then go counterclockwise along the semicircle with radius a, ending at $(-a,0)$ and go along the x-axis to $(-1,0)$.

(4) If C is any closed loop in the plane, show that $\oint_C y\cos xy\,dx + x\cos xy\,dy = 0$.

(5) For the force field $\mathbf{F} = (y+z)\mathbf{i} - (x+z)\mathbf{j} + (x+y)\mathbf{k}$, find the work done in moving a particle around the circle $x^2+y^2=1, z=0$ moving counterclockwise.

(6) Show that the electric field $\mathbf{E} = q\frac{\mathbf{r}}{r^3}$ is conservative and find a scalar potential ϕ with $\mathbf{E} = -\boldsymbol{\nabla}\phi$.

(7) Calculate $\oint 2ydx - 3xdy$ around the square with vertices $(3,1),(5,1),(5,3)$ and $(3,3)$, without integration.