Homework 8, Math 308, due April 5th

(1) Calculate $\int_\tau \boldsymbol{\nabla} \cdot \mathbf{F}\, d\tau$ where τ is the solid $x^2 + y^2 + z^2 \leq 25$ and $\mathbf{F} = (x^2 + y^2 + z^2)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$.

(2) In a charged solid in eqilibrium, one has $\mathbf{E}$, the electric field is zero at any point in the interior of the solid. Show that then all the charge must reside on the surface of the solid.

(3) Deduce Coulomb's law using Gauss' law.

(4) Calculate $\int_\sigma \mathbf{V} \cdot \mathbf{n}\, d\sigma$ over the surface of the cylindrical can bounded by $x^2 + y^2 = 9$, $z = 0$ and $z = 5$, if $\mathbf{V} = 2xy\mathbf{i} - y^2\mathbf{j} + (z + xy)\mathbf{k}$.

(5) Do Problem 17 (a), section 11 from the book.

(6) Decide which of the following vector fields are curl of another vector field and if it is, solve for it.
   (a) $-\mathbf{k}$.
   (b) $x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$.
   (c) $(y + z)\mathbf{i} + (x - z)\mathbf{j} + (x^2 + y^2)\mathbf{k}$.