Homework 9, Math 308, due April 12th

(1) Calculate the Fourier coefficients a_k, b_k for the periodic function, $f(x) = \pi + x, -\pi < x < 0$, $f(x) = \pi - x, 0 < x < \pi$. Calculate the sum of the series,

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Deduce the sum of the series $\zeta(2) = \sum_{k=1}^{\infty} \frac{1}{k^2}$.

(2) Calculate the Fourier coefficients in the exponential Fourier series for the function, $f(x) = 1 + x, -\pi < x < \pi$. Calculate the sum of the series,

$$1 - \frac{1}{3} + \frac{1}{5} - \cdots$$

(3) If $f(x) = \frac{a_0}{2} + \sum_1^{\infty} a_k \cos kx + \sum_1^{\infty} b_k \sin kx = \sum_{-\infty}^{\infty} c_n e^{inx}$, caculate c_n, c_{-n} in terms of a_n, b_n and a_n, b_n in terms of c_n, c_{-n}.

(4) Calculate the Forier transforms of the functions $f(x) = 1, 0 < x < 1$, zero elsewhere and the function $f(x) = x, 0 < x < 1$ and zero elsewhere.

(5) Find the Fourier tramsform of $f(x) = e^{-|x|}$. Verify Parseval's theorem in this case.

(6) Calculate the Fourier transform of $f(x) = \cos x, -\pi/2 < x < \pi/2$ and zero elsewhere. Use this to evaluate

$$\int_0^{\infty} \frac{\cos^2(x\pi/2)}{(1-x^2)^2} dx.$$