Nov 2

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Sec 4.8

(6)

$y_{k+2} + 25 y_k = 0$. Show that

$5^k \cos \frac{k\pi}{2}$, $5^k \sin \frac{k\pi}{2}$ form a basis

for the solution set.

Theorem 17 says that the solution set is a 2-dim. vector space.

It is easy to check that the above are solutions. So we only need to check that these are linearly independent.

If $c_1 5^k \cos \frac{k\pi}{2} + c_2 5^k \sin \frac{k\pi}{2} = 0$

for all k: we look at $k = 0, 1$, to

get $c_1 \cos 0 + c_2 \cdot \sin 0 = 0$.

$\cos 0 = 1$, $\sin 0 = 0$. So $c_1 = 0$.

$k=1$: $\quad c_1 \cdot 0 + c_2 \cdot 1 = 0$. So $c_2 = 0$.

So they are linearly indep.

(16) Find a basis for the solution space

for $y_{k+2} - 7y_{k+1} + 12y_k = 0$.

We know by Thm 17, that the solution space is 2-dimensional.

We look at the auxiliary equation.

$$r^2 - 7r + 12 = 0.$$

$(r-4)(r-3) = 0$. So $r = 3, 4$ are solutions. Then

$y_k = 3^k, \; 4^k$ are solutions

of our eqn. As in the previous problem, one checks that they are l. indep. and thus form a basis for the solution space.

(26) Show that $y_k = 1+k$ is a solution

of $y \quad - 8y \quad + 15y = 2 + 8k$.

of $y_{k+2} - 8y_{k+1} + 15y_k = 2+8k$.

and find the general solution.

If $y_k = 1+k$, we get,

$y_{k+2} - 8y_{k+1} + 15y_k = 3+k - 8(2+k)$

$+ 15(1+k) = (3-16+15) + (k-8k+15k)$

$= 2+8k$. Thus $y_k = 1+k$ is a solution.

Next we solve the homogeneous eqn. $y_{k+2} - 8y_{k+1} + 15y_k = 0$.

The auxiliary eqn. is,

$$r^2 - 8r + 15 = 0.$$

$$(r-5)(r-3) = 0.$$

So $\{5^k\}$ & $\{3^k\}$ are solutions.

Easy to check they are l. indep. So they form a basis for the solution space. Thus the general solution is

$$y_k = 1+k + c_1 3^k + c_2 5^k, \quad c_1, c_2 \in \mathbb{R}.$$

$y_k = 1 + k + c_1$ [illegible] $+ c_2$ [illegible] , $c_1, c_2 \in \mathbb{R}$.

—— x ——

See 4.9.

⑥ Find the steady state vector for $P = \begin{bmatrix} .8 & .5 \\ .2 & .5 \end{bmatrix}$.

We solve the homog. eqns given by $(P - I)\bar{x} = 0$.

$$P - I = \begin{bmatrix} -.2 & .5 \\ .2 & -.5 \end{bmatrix}.$$

$$10(P - I) = \begin{bmatrix} -2 & 5 \\ 2 & -5 \end{bmatrix} = Q.$$

$Q \rightsquigarrow \begin{bmatrix} -2 & 5 \\ 0 & 0 \end{bmatrix}$. So x_2 is free

and $2x_1 = 5x_2$. $x_1 = \frac{5}{2} x_2$.

Taking $x_2 = 2$, a solution is $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$. So the steady state vector is, $\begin{bmatrix} 5/7 \\ 2/7 \end{bmatrix}$.

—— x

<u>Sec 5.1</u>

(2) Is $\lambda = -2$ an eigenvalue of $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$?

<u>Sol 1.</u> $\lambda = -2$ is an eigenvalue means, $A\vec{x} = -2\vec{x}$ has a non-zero solution. i.e. $(A+2I)\vec{x} = \vec{0}$ has a non-zero solution, which means $\det(A+2I) = 0$. So we compute:

$$A + 2I = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}.$$

$$\det(A+2I) = 9 - 3\times 3 = 0.$$

So $\lambda = -2$ is an eigenvalue.

(10) Find a basis for the eigen space corresponding to $\lambda = 4$ for $A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$.

$$A - \lambda I = A - 4I = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}.$$

$A - \lambda I = A - 4I = \begin{bmatrix} & \\ 4 & -6 \end{bmatrix}$.

I will leave you to check that $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is a basis for the eigen-space.

(22) True or False:

a) If $Ax = \lambda x$ for some scalar λ, x is an eigen vector of A.

False. If $\lambda = 0$, then $Ax = 0$ has $x = 0$ as a solution. But eigen vectors are always non-zero.

b) If v_1, v_2 are linearly indep. eigenvectors, they correspond to distinct eigenvalues.

False. Take $A = I_{2\times 2}$, 2×2 id. matrix. Then it has only one eigen value 1, but $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (any non-zero vector for that matter) are linearly indep. eigen vectors.

c) A steady state vector for a stochastic matrix is actually an eigen vector.

True. Steady state vector is a probability vector, so non-zero.

It is an eigen vector for the eigen value 1.

d) The eigen values of a matrix are on its main diagonal.

False. As an example take $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Then the eigen values are ± 1.

[Calculate λ for $\det(A - \lambda I) = 0$:

$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$. So $\det = \lambda^2 - 1$.

Its roots are $\lambda = \pm 1$.]

e) An eigenspace of A is a null space of certain matrix.

True. The eigenspace for eigen value λ, is the null space of $A - \lambda I$.

5.2. (10) Find the characteristic polynomial of $A = \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

[I prefer $\det(\lambda I - A)$, ~~book~~ uses $\det(A - \lambda I)$.]

$$\lambda I - A = \begin{bmatrix} \lambda & -3 & -1 \\ -3 & \lambda & -2 \\ -1 & -2 & \lambda \end{bmatrix}.$$

So
$$\det(\lambda I - A) = \lambda \begin{vmatrix} \lambda & -2 \\ -2 & \lambda \end{vmatrix} + 3 \begin{vmatrix} -3 & -2 \\ -1 & \lambda \end{vmatrix} - 1 \begin{vmatrix} -3 & \lambda \\ -1 & -2 \end{vmatrix}$$

$$= \lambda(\lambda^2 - 4) + 3(-3\lambda - 2) - 1(6 + \lambda)$$

$$= \lambda^3 - 14\lambda - 12.$$

Sec 5.3

(4)
$$\begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}^k = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 4 - 3\cdot 2^k & 12\cdot 2^k - 12 \\ 1 - 2^k & 4\cdot 2^k - 3 \end{bmatrix}$$

(8) Di... $\begin{bmatrix} 5 & 1 \end{bmatrix}$ ...

(8) Diagonalize $\begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$ if possible.

It has only one eigenvalue (it is upper triangular) 5.

If $\begin{bmatrix} x \\ y \end{bmatrix}$ is an eigen vector for 5, we have $\begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5x+y \\ 5y \end{bmatrix} = \begin{bmatrix} 5x \\ 5y \end{bmatrix}$.

The first says $y=0$ & x is free

So the eigen space is spanned by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & since it does not span $\mathbb{R}^2$, the matrix cannot be diagonalized.