Oct 5

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Sec 2.5

(8) $A = \begin{bmatrix} 6 & 9 \\ 4 & 5 \end{bmatrix}$. Find the LU-factorization

So L and U will be 2 x 2 matrices.

$L = \begin{bmatrix} 1 & 0 \\ * & 1 \end{bmatrix}$.

So $L = \begin{bmatrix} 1 & 0 \\ 2/3 & 1 \end{bmatrix}$

$$A \sim \begin{bmatrix} 6 & 9 \\ 0 & -1 \end{bmatrix} = U$$

$$LU = \begin{bmatrix} 1 & 0 \\ 2/3 & 1 \end{bmatrix} \begin{bmatrix} 6 & 9 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 4 & 5 \end{bmatrix}.$$

(10) $A = \begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$ Find LU factorization.

First column of L is $\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} = \frac{-1}{5} \begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix}$.

$$A \sim \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 10 & 14 \end{bmatrix} = A_1$$

So second column of L is $\begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$

$$A_1 \sim \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$$ 3^{rd} col. of $L = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$. We verify the answer is correct.

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix}\begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}.$$

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Sec 2.8

(6) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 4 \\ -7 \\ 9 \\ 7 \end{bmatrix}$; $\vec{v}_3 = \begin{bmatrix} 5 \\ -8 \\ 6 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 7 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} -4 \\ 10 \\ -7 \\ -5 \end{bmatrix}$. Determine if $\vec{u}$ is in the subspace generated by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

We want to know whether we can solve $\vec{u} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3$. i.e.

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{u}.$$

So we look at, as usual, the augmented matrix

$$A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{u} \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{bmatrix}$$

and row-reduce:

$$A \sim \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & -14 & 9 \\ 0 & -5 & -10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 23 \\ 0 & 0 & 0 & 17 \end{bmatrix}$$

System is consistent (by 3^{rd} or 4^{th} row) and so $\vec{u}$ is NOT in the subspace generated by $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

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(10) Is $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ in the Null space of

$-2 \quad 0 \,] \quad 2$

$$A = \begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix} \ ?$$

We calculate $A \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

So it is.

(24) Find a basis for Col A and Nul A, where $A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

From the echelon form, we have two pivots 1 in the first col. and 4 in the 3^{rd} column.

So for $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in Null space, we have two eqns:

$$4x_3 + 5x_4 = 0$$

$$x_1 - 3x_2 + 6x_3 + 9x_4 = 0.$$

So $x_3 = -5/4\, x_4$;

$x_1 = 3x_2 - 6x_3 - 9x_4 = 3x_2 + \frac{15}{2}x_4 - 9x_4$

$= 3x_2 - \frac{3}{2}x_4.$

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3x_2 - \frac{3}{2}x_4 \\ x_2 \\ -5/4\, x_4 \\ x_4 \end{bmatrix}$

$= x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -\frac{3}{2} \\ 0 \\ -5/4 \\ 1 \end{bmatrix}.$ So a

basis for the Null space

is $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 0 \\ -5/4 \\ 1 \end{bmatrix} \right\}.$

A basis for Column space

is $\left\{ \begin{bmatrix} -3 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix} \right\}$, the columns

corresponding to the pivot columns.

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Sec 2.9

④ $\vec{b}_1 = [\quad$ $\vec{b}_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$.

Given $\vec{x}$ is in the span of the

Given $\vec{x}$ is in the span of linearly indep. vectors $\{\vec{b_1}, \vec{b_2}\} = \mathcal{B}$

Calculate $[\vec{x}]_{\mathcal{B}}$.

$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \vec{c}$ means, $\vec{x} = c_1 \vec{b_1} + c_2 \vec{b_2}$

So we are solving the equation,

$[\vec{b_1} \ \vec{b_2}] \ \vec{c} = [\vec{x}]$. We write the augmented matrix,

$$\begin{bmatrix} 1 & -3 & -7 \\ -3 & 5 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -7 \\ 0 & -4 & -16 \end{bmatrix}.$$

so $-4c_2 = -16$, $\boxed{c_2 = 4}$

$c_1 - 3c_2 = -7$, $c_1 - 12 = -7$, $\boxed{c_1 = 5.}$

So $\vec{c} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} = [\vec{x}]_{\mathcal{B}}$.

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10 $A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find bases for Col A and Nul A.

The pivot colns. are 1^{st}, 2^{nd} & 4^{th}.
So the 1^{st}, 2^{nd} & 4^{th} colum of A form a basis for Col A. So a basis is,

$$\begin{bmatrix} 1 \\ 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix}.$$

Nul A = Null space of the echelon form. This gives the following equations:

$$x_1 - 2x_2 + 9x_3 + 5x_4 + 4x_5 = 0$$

$$x_2 - 3x_3 \qquad - 7x_5 = 0$$

$$x_4 - 2x_5 = 0.$$

$\therefore$ $x_4 = 2x_5$; $x_2 = 3x_3 + 7x_5$.

$$x_1 = 2x_2 - 9x_3 - 5x_4 - 4x_5$$

$$= 6x_3 + 14x_5 - 9x_3 - 10x_5 - 4x_5.$$

$= -3x_3$. Thus the solutions are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ 3x_3+7x_5 \\ x_3 \\ 2x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

Thus Nul A has basis,

$$\begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

Sec 3.1 (2) Calculate the determinant

$$\begin{vmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 2 & 3 & 1 \end{vmatrix}$$ using cofactor expansion

using 1^{st} row.

$a_{11}=0$, $a_{12}=4$, $a_{13}=1$. So the determinant

is $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$,

where C_{ij} is the cofactors..

$$C_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} = -3\times 1 - 3\times 0 = -3$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 0 \\ 2 & 1 \end{vmatrix} = -5.$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & -3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 & 1 \end{vmatrix}$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & -3 \\ 2 & 3 \end{vmatrix} = 21.$$

So $\det = 0 \times (-3) + 4 \times (-5) + 1 \times 21$

$= 1$

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(12) Calculate the determinant

of $A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{pmatrix}$.

If we use the first row, clearly only the cofactor C_{11} appears, since $a_{12} = a_{13} = a_{14} = 0$. So,

$$\det A = 3 \begin{vmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix}$$

Again, $\begin{vmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = -2 \begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix}$.

The last is just $3 \times (-3)$.

So the determinant is,

$$3 \times (-2) \times 3 \times (-3) = 54.$$

(Determinant of an upper (or lower) triangular matrix is just the product of entries in the diagonal.)