Sep 28

Thursday, September 17, 2020 2:08 PM

See 2.1

6. $A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$.

Calculate AB.

$B = [\vec{b_1} \ \ \vec{b_2}]$ where $\vec{b}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and

$b_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

So $A\vec{b}_1 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \times 1 - 2 \times 2 \\ -3 \times 1 + 0 \times 2 \\ 3 \times 1 + 5 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix}$

$A\vec{b}_2 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \times 3 + (-2)(-1) \\ -3 \times 3 + 0 \times (-1) \\ 3 \times 3 + 5 \times (-1) \end{bmatrix}$

$= \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}$. So $AB = [A\vec{b}_1 \ \ A\vec{b}_2]$

$= \begin{bmatrix} 0 & 14 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}.$$

The row-column rule is similarly straight forward

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(17) If $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ and

$AB = \begin{bmatrix} -1 & 2 & -1 \\ 6 & 9 & 3 \end{bmatrix}$, determine

the first & second column of B.

Since A is a 2×2 matrix, notice that B must be $2 \times p$ matrix (only way AB will make sense) and since AB is a 2×3 matrix, $p = 3$.

So write $B = [\vec{b}_1 \ \vec{b}_2 \ \vec{b}_3]$, where $\vec{b}_i$ s are 2-vectors. We will calculate $\vec{b}_1$, calculation of $\vec{b}_2$ being similar. We have, $A\vec{b}_i$ the columns of AB so $A\vec{b}_1 = \begin{bmatrix} -1 \\ \end{bmatrix}$

[illegible] we have, [illegible] the columns

of AB so, $A\vec{b}_1 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

So $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \vec{b}_1 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

Writing the augmented matrix,

$$\begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 4 \end{bmatrix}.$$

So $\boxed{x_2 = 4}$ and $x_1 - 8 = -1$

So $\boxed{x_1 = 7}$. Thus $\vec{b}_1 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$.

Sec 2.2

8) Show that if A is invertible and $AD = I$, then $D = A^{-1}$.

A is invertible means, there is another (same size) matrix B such that $AB = I = BA$.

If $AD = I$, multiply by B to get $B(AD) = BI = B$.

But $B(AD) = (BA)D = ID = D$.

So $B = D$.

30 Find the inverse of $\begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$ if it exists.

For a 2×2 matrix, it has an inverse if and only if its determinant is non-zero.

In this case the determinant is

$$5 \times 7 - 4 \times 10 = 35 - 40 = -5 \neq 0.$$

So by Theorem 4, the inverse is

$$-\frac{1}{5}\begin{bmatrix} 7 & -10 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{7}{5} & 2 \\ \frac{4}{5} & -1 \end{bmatrix}$$

You can use the algorithm

described in the book which we will do for the next problem.

32 Find the inverse of $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ if it exists.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & -2 & 0 & 1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 6 & -2 & 1 \end{array}\right].$$

Since $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ has only 2 pivots, we cannot make it identity by row operations. So the matrix we started with is not invertible.

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See 2.3

(6) Is $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ invertible?

$$\left[\begin{array}{ccc|ccc} 1 & -5 & -4 & 1 & 0 & 0 \\ 0 & 3 & 4 & 0 & 1 & 0 \\ -3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & -5 & -4 & 1 & 0 & 0 \\ 0 & 3 & 4 & 0 & 1 & 0 \\ 0 & -9 & -12 & 3 & 0 & 1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & -5 & -4 & 1 & 0 & 0 \\ 0 & 3 & 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -3 & 1 \end{array}\right]$$

As before $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ has only 2 pivots, so not invertible.

(34) $T:\mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$. Show that T is invertible & find its inverse.

The associated matrix is:

$$\begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix}$$ whose determinant

is $6 \times 7 - (-5) \times (-8) = 42 - 40 = 2 \neq 0$

so invertible and its inverse is

$$\frac{1}{2}\begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 7/2 & 4 \\ 5/2 & 3 \end{bmatrix}.$$

So $T^{-1}(x_1, x_2) = \left(\frac{7}{2}x_1 + 4x_2, \frac{5}{2}x_1 + 3x_2\right)$