Oct 12

Saturday, October 10, 2020 11:17 AM

Sec 3.2

Say why the following are true.

(1) $\begin{vmatrix} 0 & 5 & -2 \\ 1 & -3 & 6 \\ 4 & -1 & 8 \end{vmatrix} = -1 \begin{vmatrix} 1 & -3 & 6 \\ 0 & 5 & -2 \\ 4 & -1 & 8 \end{vmatrix}$.

1^{st} & 2^{nd} row are flipped.

(2) $\begin{vmatrix} 1 & 2 & 2 \\ 0 & 3 & -4 \\ 3 & 7 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 3 & -4 \\ 0 & 1 & -2 \end{vmatrix}$

3×1^{st} row is subtracted from the 3^{rd} row.

(3) $\begin{vmatrix} 3 & -6 & 9 \\ 3 & 5 & -5 \\ 1 & 3 & 3 \end{vmatrix} = 3 \begin{vmatrix} 1 & -2 & 3 \\ 3 & 5 & -5 \\ 1 & 3 & 3 \end{vmatrix}$

The left side is just obtained by multiplying the 1^{st} row of the right side matrix by 3.

(8) Calculate,

$$\begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{vmatrix}$$

$$\rightsquigarrow \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & 5 \end{vmatrix} \rightsquigarrow \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & -10 \end{vmatrix} \rightsquigarrow \begin{vmatrix} 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 1 & -5 \end{vmatrix}$$

$$\rightsquigarrow - \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 10 \end{vmatrix} = -10$$

(24) Use determinant to decide if $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ -2 \end{bmatrix}$ are linearly independent.

They are linearly independent if and only if

$$\begin{vmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 2 & 7 & -2 \end{vmatrix} \neq 0.$$

So $$\begin{vmatrix} 2 & 7 & -2 \\ 6 & 0 & -5 \\ 4 & -7 & -3 \end{vmatrix} \neq 0 ?$$

$$\rightsquigarrow \begin{vmatrix} 2 & 7 & 2 \\ 0 & -21 & 1 \\ 0 & -21 & 1 \end{vmatrix} \neq 0 ?$$

$$\rightsquigarrow \begin{vmatrix} 2 & 7 & 2 \\ 0 & -21 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 2 \times (-21) \times 0 = 0.$$

... dependent.

So they are linearly dependent.

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Sec 3.3

(2) Use Cramers rule to calculate the solution of:

$4x_1 + x_2 = 6; \quad 3x_1 + 2x_2 = 7.$

$A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$. We wish to solve

$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \vec{b}$. First we calculate

$\det A = 4 \times 2 - 3 \times 1 = 5 \neq 0$. So A is invertible and thus we can apply Cramer's rule.

$\det A_1(\vec{b}) = \begin{vmatrix} 6 & 1 \\ 7 & 2 \end{vmatrix} = 5$

$\det A_2(\vec{b}) = \begin{vmatrix} 4 & 6 \\ 3 & 7 \end{vmatrix} = 10.$

Thus $x_1 = 5/5 = 1, \quad x_2 = 10/5 = 2.$

So the solution is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(12) Compute the adjugate of

$A = \begin{bmatrix} 1 & 1 & 3 \\ \ldots \end{bmatrix}$ and find the

$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and find the inverse using Thm 8.

$C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$ $\quad C_{21} = -\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 2$

$C_{12} = -\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} = 2$ $\quad C_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1$

$C_{13} = \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} = -2$ $\quad C_{23} = -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$

$C_{31} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5$ $\quad C_{32} = -\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = -7$

$C_{33} = \begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} = 4$

So determinant is:

$a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} = 1 \times 1 + 1 \times 2 + 3 \times (-2)$
$= 3 - 6 = -3 \neq 0.$

So A is invertible, the adjugate is

$B = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -7 \\ -2 & -1 & 4 \end{bmatrix}$ and the inverse

is $-\frac{1}{3} B$.

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