Oct 19

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Sec 4.1 (3) $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2+y^2 \le 1 \right\}$.

H is not a subspace of $\mathbb{R}^2$.

Notice that $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in H.

But $10 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$ is not in H.

(6) All polynomials of the form $p(t) = a + t^2$, a is in $\mathbb{R}$.

It is not. These are subsets of $\mathbb{P}_n$ for any $n \geq 2$. But if this set is a subspace, we must have say $2p(t) = 2a + 2t^2$ in this sets but it is not.

(10) Let $H = \left\{ \begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix} : t \in \mathbb{R} \right\}$

Show that H is a subspace of $\mathbb{R}^3$.

$$\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix} = t \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}.$$ So H is the span of $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ and so a subspace.

(20) $C[a,b]$ = the set of all continuous functions on the closed interval $[a,b]$. Show that this is a subspace of the set of all real functions on $[a,b]$

- Basically, you need to know that if $f, g \in C[a,b]$ so is $f+g$; similarly for any scalar $\theta \in \mathbb{R}$, $\theta f \in C[a,b]$.

——— x ———

See 4.2.

(6) Find an explicit description of Nul A by finding a spanning set.

$$A = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We convert into reduced echelon form.

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 6 & -8 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So the equations give,

$$x_1 = -6x_3 + 8x_4 - x_5$$

$$x_2 = 2x_3 - x_4$$

So the solutions are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -6x_3 + 8x_4 - x_5 \\ 2x_3 - x_4 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

So $\begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Span (and are linearly indep)

Nul A.

(16) Find A such that Col A

$$= \left\{ \begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} : b, c, d \in \mathbb{R} \right\}.$$

$$\begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} = b \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -1 \\ 1 \\ 5 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix}.$$

So we can take

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$

(32) Define a linear transformation $T : P_2 \longrightarrow \mathbb{R}^2$ by $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$.

Find two vectors spanning the kernel of T and describe the range of T.

p is in the kernel of T means $T(p) = \vec{0}$. So $p(0) = 0$.

If $p(t) = a_0 + a_1 t + a_2 t^2$, $p(0) = 0$ just means $a_0 = 0$. So the kernel is spanned by $p_1 = t$ and $p_2(t) = t^2$.

Similarly the range is just $\begin{bmatrix} p(0) \\ p(0) \end{bmatrix} = p(0) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. So it is the subspace spanned by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ since $p(0)$ can take any value as p varies.

See 4.3

(6) Are $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -5 \\ 6 \end{bmatrix}$ lin. indep? Do they span $\mathbb{R}^3$?

Linear indep. means, the only solution to $x_1 \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ -5 \\ 6 \end{bmatrix} = \vec{0}$ is $x_1 = x_2 = 0$.

So we look at the aug. matrix

$$\begin{bmatrix} 1 & -4 & 0 \\ 2 & -5 & 0 \\ -3 & 6 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -4 & 0 \\ 0 & 3 & 0 \\ 0 & -6 & 0 \end{bmatrix}.$$

$$\rightsquigarrow \begin{bmatrix} 1 & -4 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ So $3x_2 = 0$

and thus $x_2 = 0$. $x_1 - 4x_2 = 0$ coupled with $x_2 = 0$, we get $x_1 = 0$

So they are linearly indep.

They span $\mathbb{R}^3$ means for any $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, we should be able to solve $x_1 \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ -5 \\ 6 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$$\begin{bmatrix} 1 & -4 & a \\ -2 & -5 & b \\ 3 & 6 & c \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & -4 & a \\ 0 & 3 & b+2a \\ 0 & -6 & c-3a \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -4 & a \\ 0 & 3 & b+2a \\ 0 & 0 & c+2b+a \end{bmatrix}$$

Last equation says, the system is inconsistent if $a + 2b + c \neq 0$.

is inconsistent if $a+2b+c \neq 0$.

In particular, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is NOT

in the span of the two vectors.

(16) Find a basis for the space

spanned by

$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}$

$\vec{v}_5 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 1 & 1 & -1 & -4 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 0 & -1 & -7 & -9 & 1 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -8 & -12 & 4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -12 & 20 \end{bmatrix}$$

The pivot columns are the first four, and so $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ form a basis for the span of $\vec{v}_1, \ldots, \vec{v}_5$.

<u>Extra</u>: Let $L(P_2, \mathbb{R}^2)$ be the set of all linear transformation from $\mathbb{P}_2$ to $\mathbb{R}^2$. We want to show that $L(P_2, \mathbb{R}^2)$ is a

vector space under the operations:

$$(T_1+T_2)(p) = T_1(p)+T_2(p) \qquad T_1, T_2 \in L$$

$$(cT)(p) = c\,T(p) \qquad T \in L, c \in \mathbb{R}.$$

All checkings are routine.

For example to $T_1+T_2 \in L$; we check properties for a linear transf.

Let $U = T_1+T_2$. Then,

$$U(p+q) = (T_1+T_2)(p+q) = T_1(p+q)+T_2(p+q) \quad \text{(by def.)}$$

$$= T_1(p)+T_1(q) + T_2(p)+T_2(q) \quad (T_1, T_2 \in L)$$

$$= \big(T_1(p)+T_2(p)\big) + \big(T_1(q)+T_2(q)\big)$$

$$= (T_1+T_2)(p) + (T_1+T_2)(q) \quad \text{by def}$$

$$= U(p)+U(q).$$

All checkings are similar & boring.

You must do this a few times to get comfortable.