Sec 1.3, 1.4

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Sec 1.3 :

Exer 12:

$\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$ $\vec{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$

$\vec{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

Determine if $\vec{b}$ is a linear combination of $\vec{a_1}$, $\vec{a_2}$ and $\vec{a_3}$.

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

Inconsistent. So $\vec{b}$ is NOT a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

<u>Exer 25</u>:

Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and $W = \text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$

a. Is $\vec{b}$ in $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$?

How many vectors are in $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

No; $\vec{b} \neq \vec{a}_1, \vec{a}_2$ or $\vec{a}_3$. There are 3 vectors in $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

b. Is $\vec{b}$ in W? How many vectors are in W?

For the second part, there are infinitely many; for example $\lambda \vec{a}_1$, λ any real # is in W, these are distinct for different λ.

For the first part we follow our usual technique.

$$\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

This system is consistent.

(Do you see why?)

———— x ————

See 1.4

Exer 22:

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$$

Does $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ span $\mathbb{R}^3$?

Spans, means, given <u>ANY</u> vector

$\vec{a} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, we should be able

to solve $x_1 \vec{v}_1 + x_2 \vec{v}_2 + x_3 \vec{v}_3 = \vec{a}$.

So as usual we look at the augmented matrix, $[\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{a}]$ and see whether we have a consistent system for ANY a, b, c.

$$\begin{bmatrix} 0 & 0 & 4 & a \\ 0 & -3 & -1 & b \\ -2 & 8 & -5 & c \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} -2 & 8 & -5 & c \\ 0 & -3 & -1 & b \\ 0 & 0 & 4 & a \end{bmatrix}$$

System is consistent: (Why?)

→ We explain why if it is not clear:

Last row gives $4x_3 = a$.

So $\boxed{x_3 = a/4}$

2nd row:

$$-3x_2 - \frac{a}{4} = b.$$

So $\boxed{x_2 = \left(b + \frac{a}{4}\right) \Big/ -3}$

1st row: $-2x_1 + 8x_2 - 5x_3 = c$

So $\boxed{x_1 = -\frac{8x_2 + 5x_3 - c}{2}}$.

Since we have solved for x_2, x_3, we get x_1.