Homework 2, Math 310, due Friday 14, September

(1) Let P be a statement. Write truth tables for the statements, $P \wedge \neg P$ and $P \vee \neg P$.
(2) Let P, Q, R be statements. Write truth tables for $(P \vee Q) \Rightarrow R$ and $(P \Rightarrow R) \wedge (Q \Rightarrow R)$. Deduce that these two statements are logically equivalent.
(3) Using the above, show that the following two statements are logically equivalent. The proof should be short, no more than a few sentences. You may assume properties of integers. Let a, b be integers.
    (a) If a or b is odd then ab is odd.
    (b) If a is odd then ab is odd and if b is odd then ab is odd.
(4) Writing an appropriate truth table, show that the following two statements are logically equivalent.
    (a) If x is a real number such that $x^2 = 4$ then $x = 2$ or $x = -2$.
    (b) If x is a real number such that $x^2 = 4$ and $x \neq 2$ then $x = -2$.
(5) Express the following sets in the set-builder notation.
    (a) $\{2, 4, 6, 8, \ldots\}$.
    (b) $\{1, 2, 4, 8, 16, \ldots\}$.
    (c) $\{\ldots, \frac{1}{25}, \frac{1}{5}, 1, 5, 25, \ldots\}$.
(6) Express the following sets in the roster method.
    (a) $\{x \in \mathbb{R} \mid \sin x = 0\}$.
    (b) $\{x \in \mathbb{Z} \mid x + 5 \geq 0\}$.
    (c) $\{x \in \mathbb{R} \mid x^2 - 3x + 2 = 0\}$.