## Homework 3, Math 310, due Monday 21, September

(1) For this problem, assume that the universal set is $\mathbb{Z}$, the set of integers. Write in the roster method, the truth sets of the following predicates.
(a) $P(x): x^{2}=1$.
(b) $P(x): x^{2}+3 x+1=0$.
(c) $P(x): x^{1 / 3} \in \mathbb{Z}$.
(2) In the following, write each statement as an English sentence (with no symbols), its negation both in English and symbolically and determine whether the statement is true or false (using your past knowledge, with justifications). The universal set is $\mathbb{Z}$.
(a) $(\exists x \in \mathbb{Z})\left(x^{2} \geq 5\right)$.
(b) $(\forall x \in \mathbb{Z})(x<x+1)$.
(c) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\left(y^{2}<x\right)$.
(3) Decide whether the following statements are true or false (using your past knowledge).
(a)

$$
\begin{aligned}
{[[(\forall x \in \mathbb{R})(\exists n \in \mathbb{N})(n>x)] \vee[(\exists x \in \mathbb{R})} & \left.\left.\left(x^{2}>1\right)\right]\right] \\
& \wedge\left[(\forall x \in \mathbb{R})\left(x^{2}+1>0\right)\right] .
\end{aligned}
$$

(b)

$$
\begin{aligned}
{\left[\left[(\exists x \in \mathbb{Q})\left(x^{2}=3\right)\right] \wedge\right.} & {\left.\left[(\exists x \in \mathbb{N})\left(x^{2}=4\right)\right]\right] } \\
& \vee\left[\left[(\exists x \in \mathbb{R})\left(x^{2}=3\right)\right] \wedge\left[(\forall x \in \mathbb{Q})\left(x^{2} \neq 4\right)\right]\right]
\end{aligned}
$$

As you can see from the last problem, that expressing statements symbolically, while precise, is somewhat opaque for daily use.

