## Homework 4, Math 310, due Monday 28, September

Clarity and logical reasoning are paramount in these problems. Let a friend read your answers and comment on it to make sure that your arguments are complete.

- (1) Give a direct proof to show that if a, b are integers which are squares of integers, then ab is the square of an integer.
- (2) Write the converse to the statement appearing in the first problem. Is the converse true or false? If true, give a proof and if false, give a counterexample.
- (3) Give a constructive proof to show that the equation,  $x^5 x^4 + x^3 x^2 + x 1 = 0$  has a solution in integers.
- (4) Using results you have studied in previous courses you have taken, show that if x, y are real numbers, then  $|xy| \leq |x^2 + y^2|$ .
- (5) Use intermediate value theorem (and thus a non-constructive proof) to show the following: A hiker walks up a hill starting at 6am and reaching the top at 6pm. He walks back next day from the top starting at 6am and reaching the bottom at 6pm along the same path. Show that there exists some point in his path where he was at the same time, going up or down.
- (6) Show that if a, b are postive real numbers and  $ab \neq (a+b)^2/4$ , then  $a \neq b$ .
- (7) (a) If y is an irrational number and  $x \neq 0$  is a rational number, show that xy is irrational.
  - (b) If x is a real number and y is an irrational number, show that either x + y or -x + y is irrational.
  - (c) Give an example of x, y as before, so that both x + y and -x + y are irrational.
  - (d) Give an example of x, y as before so that one of x+y, -x+y is irrational and the other is rational.
- (8) If a, b, c are integers with  $a^2 + b^2 = c^2$ , then show that either a or b is even.