Homework 6, Math 310, due October 12th, Monday

(1) As usual, define $n! = 1\cdot 2\cdot 3\cdots n$ where n is any natural number. By convention $0!$ is defined to be 1. Similarly $\binom{n}{r}$ for $0 \le r \le n$ is defined to be,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(a) Show that

$$\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1}$$

(b) Prove the binomial theorem,

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}xy^{n-1} + y^n$$

(2) Let $\mathbb{Z}$, the set of integers be the universal set. Write using roster method the following sets.

(a) $A = \{a \in \mathbb{Z} \mid a^2 < 7\}$.

(b) $B = \{a \in \mathbb{Z} \mid \exists b \in \mathbb{Z}, ab = 1\}$.

(c) $C = \{a \in \mathbb{Z} \mid a = 3b+1, b \in \mathbb{Z}\}$.

(3) Let the notation be as in the previous problem. Write using roster method the following sets.

(a) $B \cup C$

(b) $A - C$

(c) $A \cap B \cap C$

(4) Prove that for any three sets $P, Q, R \subseteq \Omega$, where Ω is the universal set, $P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$ and $(P \cup Q)^c = P^c \cap Q^c$.

(5) Let $S_1, S_2, \ldots, S_n$ be finite sets.

(a) Show that $\cup_{i=1}^n S_i$ is a finite set, where the above notation is the short form for $S_1 \cup S_2 \cup \cdots \cup S_n$.

(b) Show that $|\cup_{i=1}^n S_i| \le \sum_{i=1}^n |S_i|$.

(c) If $S_i \cap S_j = \emptyset$ for all $i \ne j$, show that $|\cup_{i=1}^n S_i| = \sum_{i=1}^n |S_i|$.