Homework 8, Math 310, November 2nd

(1) Define a relation on $\mathbb{R}$ as follows. Two real numbers x, y are equivalent if $x - y \in \mathbb{Z}$. Show that this is an equivalence relation. Show that the set of equivalence classes of this relation is (naturally) bijective to the set of points on the unit circle.
(2) Define a relation on $P = \mathbb{R}^2 - \{(0,0)\}$ as follows. If $(a,b), (c,d) \in P$, they are related if for some positive real number α, $a = \alpha c$ and $b = \alpha d$. Show that this is an equivalence realtion and the set of equivalence classes is (naturally) bijective to the points on a unit circle.
(3) Define a relation on $\mathbb{R}[x]$, the set of polynomials as follows. Two polynomials $f(x), g(x)$ are related if $f(0) = g(0)$. Show that this is an equivalence relation and the set of equivalence classes is (naturally) bijective to $\mathbb{R}$.
(4) Let $C(\mathbb{R})$ be the set of all continuous functions on $\mathbb{R}$. If $f, g \in C(\mathbb{R})$ we say that $f \sim g$ if $\int_0^1 f dx = \int_0^1 g dx$. Show that this is an equivalence realtion. Can you identify the set of equivalence classes with a familiar set?
(5) Define a realtion on $\mathbb{N} \times \mathbb{N}$ by declaring that (a,b) is related to (c,d) if $a + d = b + c$. Show that this is an equivalence realtion and the set of equivalence classes is (naturally) bijective to $\mathbb{Z}$.
(6) Let $P = \mathbb{Z} \times (\mathbb{Z} - \{0\})$, the set of pairs of integers (a,b) with $b \neq 0$. Define a relation on P by declaring that for $(a,b), (c,d) \in P$, $(a,b) \sim (c,d)$ if $ad = bc$. Show that this is an equivalence realtion and the set of equivalence classes is (naturally) bijective to $\mathbb{Q}$.
(7) Define a function $\phi : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ as follows.
$$\phi(m,n) = \frac{(m+n-2)(m+n-1)}{2} + n.$$
Show that ϕ is a bijection. (A suggestion: Plot the points $(m,n) \in \mathbb{N} \times \mathbb{N}$ and look at $\phi(m,n)$ for small values of m, n).