

Math 310, Homework 3, due 26th September 2011

- (1) Write each of the following statement in English that does not use quantifier symbols. Write the negation of each of these in symbolic form without using the negation symbol \neg .
- (a) $(\exists x \in \mathbb{Q})(x > \sqrt{2})$.
 - (b) $(\forall x \in \mathbb{Q})(x^2 - 2 \neq 0)$.
 - (c) $(\forall x \in \mathbb{Z})(x \text{ is even or } x \text{ is odd})$.
 - (d) $(\exists x \in \mathbb{R})(\cos(2x) = 2 \cos x)$.
- (2) Decide which of the following are true where the universe of discourse is the set of integers. Write the negation of each of these without the symbol \neg .
- (a) $(\exists m)(\exists n)(m > n)$.
 - (b) $(\exists m)(\forall n)(m > n)$.
 - (c) $(\forall m)(\exists n)(m > n)$.
 - (d) $(\forall m)(\forall n)(m > n)$.
 - (e) $(\exists n)(\forall m)(m^2 \geq n)$.
 - (f) $(\forall n)(\exists m)(m^2 \geq n)$.
- (3) Give examples (called *counterexamples*) to show the following.
- (a) $\exists x(P(x) \wedge Q(x))$ is not equivalent to $(\exists xP(x)) \wedge (\exists xQ(x))$.
 - (b) $\forall x(P(x) \vee Q(x))$ is not equivalent to $(\forall xP(x)) \vee (\forall xQ(x))$.