

Homework 4, Math 310, due October 3rd, 2011

Below are some exercises in writing proofs. Write them in English. In particular, do not use the symbols \wedge, \vee, \neg . You may use other symbols as you see fit, but try not to overdo them, since it will make it harder to read. On questions about real numbers, you are allowed to use all the standard properties that you know, but if you feel it is not standard, either prove them or quote the result explicitly to make the proof complete. Use your judgement.

- (1) Prove that there exists a set X such that for any set A , $A \cup X = A$.
- (2) If A, B, C are sets, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- (3) Prove that for all $x \in \mathbb{R}$, $x^2 + x + 1 > 0$.
- (4) Given $\epsilon > 0$ show that there exists a $\delta > 0$ such that if $|x-3| < \delta$ then $|x^2 - 9| < \epsilon$.
- (5) Define a sequence $\{s_n\}$ by,

$$s_n = \sum_{k=0}^n \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}.$$

Prove that given $\epsilon > 0$, there exists an N so that for all $n, m \geq N$, $|s_n - s_m| < \epsilon$. (Hint: Use the fact that $s_n = 2(1 - \frac{1}{2^{n+1}})$)