Homework 7, Math 310, due October 24th, 2011

For this homework, you are allowed to use only facts we have proved from set theory.

(1) If A, B are sets, prove that a subset $\Gamma \subset A \times B$ is the graph of some function from A to B if and only if the first projection $p : \Gamma \to A$ is a bijection.

(2) If $\Gamma \subset A \times B$ is the graph of a function $f : A \to B$, prove that f is injective if and only if $q : \Gamma \to B$, the second projection is injective. Similarly, prove that f is surjective if and only if $q : \Gamma \to B$ is surjective.

(3) Let $f : A \to B$, $g : B \to C$ be functions where A, B, C are sets. Consider $\Gamma_f \subset A \times B$, the graph of f, $\Gamma_g \subset B \times C$, the graph of g. Now consider the sets $\Gamma_f \times C \subset A \times B \times C$ and $A \times \Gamma_g \subset A \times B \times C$. Let $\Gamma = \theta(\Gamma_f \times C \cap A \times \Gamma_g) \subset A \times C$ where $\theta : A \times B \times C \to A \times C$ is the projection defined as $\theta((a, b, c)) = (a, c)$. Show that Γ is the graph of $g \circ f$.

(4) Let $f : A \to B$ be a bijection and let $f^{-1} : B \to A$ its inverse. If $\Gamma \subset A \times B$ is the graph of f, show that the set, $\Gamma' \subset B \times A$ defined as $\Gamma' = \{(b, a) | (a, b) \in \Gamma\}$ is the graph of f^{-1}.

(5) Let $f : A \to B$ be a surjection and let $g : B \to C$ be an injection. If $g \circ f$ is a bijection, show that f, g are bijections.