Homework 10, Math 310, due November 12th, 2012

(1) If $x, y \in \mathbb{N}$ and $x^2 > y^2$, prove that $x > y$.

(2) Let $a > 1$ be a natural number. Given $b \in \mathbb{N}$, prove that there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, $a^n \geq b^2 + b$.

(3) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ be defined as $f(a, b) = (a+1, b(a+1))$. Consider the function assured by the universal property, $\phi : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$, where $\phi(1) = (1, 1)$ and $\phi(n + 1) = f(\phi(n))$. As in last week's home work, let $\psi : \mathbb{N} \to \mathbb{N}$ be defined as $\psi(n) = \pi(\phi(n))$ for all $n \in \mathbb{N}$, where $\pi : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is the second projection. Prove that $\psi(n+1) = (n+1)\psi(n)$. (You should be able to recognize that $\psi(n) = n!$, the factorial function).

(4) Prove that if $a_1, \ldots, a_n \in \mathbb{Z}$, then

$$|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| + \cdots + |a_n|.$$

(5) Prove that if $x, y \in \mathbb{Z}$, then $|x - y| \geq ||x| - |y||$.