Math 310, Homework 3, due 24th September 2012

(1) Write each of the following statement in English that does not use quantifier symbols. Write the negation of each of these in symbolic form without using the negation symbol $\neg$.
   (a) $(\exists x \in \mathbb{Q})(x > \sqrt{2})$.
   (b) $(\forall x \in \mathbb{Q})(x^2 - 2 \neq 0)$.
   (c) $(\forall x \in \mathbb{Z})(x \text{ is even or } x \text{ is odd})$.
   (d) $(\exists x \in \mathbb{R})(\cos(2x) = 2\cos x)$.

(2) Decide which of the following are true where the universe of discourse is the set of integers. Write the negation of each of these without the symbol $\neg$.
   (a) $(\exists m)(\exists n)(m > n)$.
   (b) $(\exists m)(\forall n)(m > n)$.
   (c) $(\forall m)(\exists n)(m > n)$.
   (d) $(\forall m)(\forall n)(m > n)$.
   (e) $(\exists n)(\forall m)(m^2 \geq n)$.
   (f) $(\forall n)(\exists m)(m^2 \geq n)$.

(3) Give examples (called *counterexamples*) to show the following.
   (a) $\exists x(P(x) \wedge Q(x))$ is not equivalent to $(\exists x P(x)) \wedge (\exists x Q(x))$.
   (b) $\forall x(P(x) \vee Q(x))$ is not equivalent to $(\forall x P(x)) \vee (\forall x Q(x))$.