Math 310, Homework 6, due 15th October 2012

You must justify all your answers, by giving a proof of your answer. You may use any property of numbers you have studied.

(1) We saw in class that if $f : A \to B, g : B \to C$ are functions and $g \circ f$ is injective, so is f. Give an example to show that under the above hypothesis, g need not be injective.

(2) Let A, B be non-empty sets. Denote by S, the set of *all* functions from A to B. We have a function $F : S \times A \to B$, given as $F((f, a)) = f(a)$, where $f \in S, a \in A$. Prove that F is surjective. Under what conditions on A, B will F be injective?

(3) Decide which of the following are equivalence relations.
  (a) Relation on $\mathbb{Z}$ defined as, $a \sim b$, $a, b \in \mathbb{Z}$ if $a - b$ is even.
  (b) Relation on $\mathbb{Z}$ defined as $a \sim b$, $a, b \in \mathbb{Z}$ if $a - b$ is odd.
  (c) Let C be the set of all circles in the plane. Define $s \sim t$ for $s, t \in C$ if they have the same radius.

(4) Let $A = \{a, b, c\}$, a set with three distinct elements. Write down all possible subsets $R \subset A \times A$ which are equivalence relations.

(5) Let $R \subset S \times S$ be an equivalence relation. If $T \subset S$, prove that $R \cap T \times T$ gives an equivalence relation on T.

(6) In problems 3 a) and 4 above, describe the set of all equivalence classes.

(7) Consider the equivalence relation (you do not have to check it is) on $\mathbb{R}$ given by, for $x, y \in \mathbb{R}$, $x \sim y$ if $x = y + 2\pi n$ for some $n \in \mathbb{Z}$. Describe a surjective map $f : \mathbb{R} \to S^1$, where S^1 is the unit circle (i. e. circle in the plane with center the origin and radius 1), so that the above equivalence relation is induced by f.