

Midterm 1, Math 310, 28, September, 2012, 10.10-11am

Please read questions carefully. Please write legibly.

- (1) Decide which of the following are mathematical statements. I expect only 'yes' or 'no' as answers. Be aware that English language is open to some interpretation and so use your judgment.
 - (a) Mathematics is easy.
 - (b) Harry Potter is a wizard.
 - (c) Have you read Rowling's latest book?
 - (d) This road is narrow or the sea is blue.
- (2) Write truth tables for the following.
 - (a) $P \Rightarrow (Q \Rightarrow R)$.
 - (b) $P \Rightarrow (\neg R \vee Q)$.
- (3) Write the negation of the following statements.
 - (a) $\forall x \in S, P(x)$.
 - (b) $\exists y \in S, Q(y)$.
 - (c) $\exists x \in S(\forall y \in T, R(x, y))$.
- (4) For the following problems, you may use all standard properties of real numbers, like addition, multiplication, associativity, distributivity, commutativity etc. But, when you use one of them, say *by such and such a property* for justification. Do not include scratch work (like writing *givens* and *goals*), do it away from the main body of your answer.
 - (a) Give a direct proof for the formula, $\forall x, y \in \mathbb{R}, (x + y)(x - y) = x^2 - y^2$.
 - (b) Write a proof by contradiction for the following statement.
If $x^2 \neq 4$, then $x \neq 2$.