Midterm 2, Math 310, November 2nd, 2012

Read questions carefully. Please write legibly.

(1) (a) Given a relation $\sim$ on a set A, state the three conditions which make it an equivalence relation.
(b) Let $A = \{a, b, c, d\}$ be a set with four distinct elements. Describe all subsets $R \subset A \times A$ which give an equivalence relation on A satisfying $a \sim d$. How many distinct equivalence classes are there in each of these cases?

(2) (a) Define injectivity for a function $f : A \to B$ for sets A, B.
(b) Prove that if $f : A \to B$ and $g : B \to C$ are injective, so is $g \circ f$.

(3) Assume Peano's axioms, addition satisfying the two properties, 1) $\sigma(m) = m + 1, \forall m \in \mathbb{N}$, 2) $m + \sigma(n) = \sigma(m + n), \forall m, n \in \mathbb{N}$ and the associative law for addition. Prove that addition is commutative.

(4) (a) Define 'greater than' ($>$) and state the well-ordering principle.
(b) If $x, y, z \in \mathbb{N}$ and $xz > yz$, prove that $x > y$. (You may use the following without proofs. The two properties of addition and multiplication, associativity and commutativity of both of them, distributivity, and of course the well-ordering principle).