Math 417, Homework 1, due 14th September 2010

(1) Using induction or otherwise, show that,

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}.$$

(2) (Division Algorithm). Let $a, b \in \mathbb{Z}$ with $b > 0$. Show that there exists unique integers q (quotient) and r (remainder) such that $a = bq + r$ with $0 \leq r < b$. (Hint: Consider the set $S = \{a - bq \geq 0 | q \in \mathbb{Z}\}$).

(3) (Greatest common divisor) Given $a, b \in \mathbb{Z}$ with at least one of them non-zero, show that there exists a positive integer d which can be written as $d = ax + by, x, y \in \mathbb{Z}$ so that d divides both a, b (that is, $a/d, b/d$ are integers) and if e divides both a, b then e divides d. (Hint: Consider the set $S = \{ax + by > 0 | x, y \in \mathbb{Z}\}$.)

In the next problems, fix an $\epsilon > 0$. You are expected to find an N (depending on ϵ) satisfying the requirement.

(4) Find an N so that for all $n > N$,

$$|\sum_{k=1}^{n} \frac{1}{k(k+1)} - 1| < \epsilon.$$

(5) Find an N so that for all $n > m > N$,

$$|\sum_{k=m}^{n} \frac{1}{k^2}| < \epsilon.$$

(6) Let $\{x_n\}$ be a sequence of real numbers with $\lim x_n = a$ and $0 \leq a < 1$. Show that $\lim x_n^n = 0$.