Math 417, Homework 11, due December 7th 2010

(1) Decide which of the following are manifolds.
   (a) The unit circle, $x^2 + y^2 = 1$ in the plane.
   (b) The curve in the plane defined by $y^2 = (x - a_1)(x - a_2)\cdots(x - a_n)$ where $a_1, a_2, \ldots, a_n$ are distinct real numbers.
   (c) The curve defined by $y^2 = x^5 + x^2$.

(2) Show that a manifold is regular and thus metrizable.

(3) Let X be a normal space and let $U_1, \ldots, U_n$ an open cover of X. If f is a continuous function on X, show that there exists continous functions f_i on X such that the support of f_i is contained in U_i for all i and $\sum f_i(x) = f(x)$ for all $x \in X$.