Math 417, Homework 8, due November 9th 2010

We assume that all spaces are Hausdorff and locally compact unless otherwise mentioned.

(1) Let X, Y be non-compact topological spaces and let X', Y' be their one-point compactifications. Let $f : X \to Y$ be a continuous function and define $g : X' \to Y'$ by $g(x) = f(x)$ if $\neq \infty$ and $g(\infty) = \infty$. (Do not confuse the symbol ∞. There are two of them, one for X' and another for Y', though we used the same symbol). Show that g is continuous if and only if f is proper.

(2) Show that the map $f : \mathbb{R} \to S^1 \subset \mathbb{R}^2$ given by

$$f(t) = \left(\frac{t^2 - 1}{t^2 + 1}, \frac{2t}{t^2 + 1} \right),$$

is a homeomorphism onto $S^1 - \{(1, 0)\}$. Thus prove that S^1 is homeomorphic to the one-point compactification of $\mathbb{R}$.

(3) Show that the closed unit square region is homeomorphic to the closed unit disc. (Same is true for open regions).

(4) Show that a continuos bijection from a compact set to a Hausdorff space is a homeomorphism.

(5) Let $f : X \times Y \to \mathbb{R}$ be continuous where Y is compact. Show that the map $g : X \to \mathbb{R}$ defined as $g(x) = \sup_{y \in Y}\{f(x, y)\}$ is continuous.