Math 417, Homework 9, due November 16th 2010

(1) Let X be second countable and let $A \subset X$ be an uncountable set. Show that there exists $a \in A$ (in fact uncountably many such as) which is a limit point of A.

(2) Let $f : X \to Y$ be a closed continuous surjective map. (Recall, a closed map means that the image of closed sets are closed) such that $f^{-1}(y)$ is compact for any $y \in Y$.
  (a) Show that if X is Hausdorff, so is Y.
  (b) Show that if X is regular, so is Y.
  (c) Show that if X is locally compact, so is Y.

(3) Show that every locally compact Hausdorff space is regular.

(4) Show that a connected normal space with more than one point is uncountable.

(5) Let $C^0([0,1])$ be the set of continuous functions on the closed interval $[0,1]$ with the sup topology. Show that the map, $\phi : C^0([0,1]) \times [0,1] \to \mathbb{R}$ given by $\phi(f,a) = f(a)$ is continuous.