Math 418, Homework 1, due February 8th, 2011

(1) A collection of subsets $\mathcal{A}$ which cover a topological space X is called *fundamental* if for any subset $U \subset X$ is open if and only if $U \cup A$ is open in A (with the subspace topology) for all $A \in \mathcal{A}$.
   (a) Show that if $\mathcal{A}$ is an open cover, it is fundamental.
   (b) Show that if $\mathcal{A}$ is a closed cover and $\mathcal{A}$ is finite, it is fundamental.
   (c) Show that if $\mathcal{A}$ is a locally finite cover by closed sets, it is fundamental.

(2) Show that a locally finite cover of a compact space is finite.

(3) Let $\{U_i\}_{i \in \mathbb{N}}$ be a locally finite open cover of $\mathbb{R}^n$. Show that there exists an open cover $\{V_i\}_{i \in \mathbb{N}}$ with $\overline{V_i} \subset U_i$ for all $i \in \mathbb{N}$.