Math 418, Homework 21, due February 15th, 2011

- (1) Show that the product of a paracompact space and a compact space is paracompact.
- (2) Show that a locally finite cover of a compact space is finite.
- (3) Prove that if a locally compact space is second countable, then it is paracompact.
- (4) Let $f: X \to Y$ be a continuous surjective map with $f^{-1}(y)$ compact for any $y \in Y$. If X, Y are Hausdorff, show that X is paracompact if and only if Y is.
- (5) Let G be a Hausdorff topological group.
 - (a) If $A, B \subset G$ are compact subsets, show that the subspace

$$AB = \{ab | a \in A, b \in B\} \subset G$$

is compact.

(b) Show that a locally compact connected topological group is paracomapct.