Math 418, Homework 3, due February 22nd, 2011

(1) (a) Give an example of a topological group G and closed subsets $A, B \subset G$ such that $AB = \{ab | a \in A, b \in B\}$ is not closed.
(b) Show that a toplogical group is Hausdorff if and onlf if the intersection of all open sets containig $e \in G$, the identity element is $\{e\}$.
(c) A continuos map $f : G \to H$ of Hausdorff toplogical groups is said to be a *homomorphism* if $f(xy) = f(x)f(y)$ for all $x, y \in G$. Show that if G is compact and the homomorphism f is surjective, then f is open.
(d) A subset $H \subset G$ of a topological group is called a subgroup if for any $x, y \in H$, $x^{-1}y \in H$. A group is abelian if $xy = yx$ for any x, y. Show that if $H \subset G$ is an abelian subgroup and G is Hausdorff, then $\overline{H}$, the closure is also an abelian subgroup.

(2) As discussed in class, let $X_i = \{0, 2\}$ and let $C = \prod_{i=1}^{\infty} X_i$ be the Cantor set.
(a) Show that a compact metric space X is second countable.
(b) With X as above, let $\{U_n\}$ be a countable basis and define for any $n \in \mathbb{N}$, $f_n(0) = \overline{U_n}$ and $f_n(2) = X - U_n$. If $\alpha = (\alpha_n) \in C$, show that $\cap_{n=1}^{\infty} f_n(\alpha_n)$ is either empty or a single point.
(c) Show that there exists a continuous surjection from C to X.