Math 418, Homework 9, due April 12th 2011

(1) Do Problem 1, page 353.
(2) Do problem 4, page 353.
(3) We say that a topological space X has the fixed point property if any cintinuous function $f : X \to X$ has a fixed point. We have seen that any closed interval or a closed disc have this property. Decide which of the following have the fixed point property.
  (a) A closed bounded rectangle.
  (b) The real plane.
  (c) An open interval.
  (d) The circle.
(4) Let $f : B^2 \to \mathbb{R}^2$ be a continuous map and let C be the boundary circle of B^2. Let $P \in \mathbb{R}^2$ be a point not in the image $f(C)$. If the map $f_* : \pi_1(C) \to \pi_1(\mathbb{R}^2 - \{P\})$ is not trivial, show that there exists a point $Q \in B^2$ such that $f(Q) = P$.
(5) Let f, C be as in the previous problem. If $f(P) \cdot P \neq 0$ for all $P \in C$, where $\cdot$ denotes the usual dot product in $\mathbb{R}^2$, show that there exists a $Q \in B^2$ with $f(Q) = 0$.